Modern Portfolio

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Chapter - 5
Modern Portfolio

*** Portfolio:**

If we invest our total investment fund in only one asset, the investment is called investment in isolation (single). But if we invest our total investment funds into more than one asset the investment is called diversification and the set or collection of assets in which investment is made is called portfolio. The only one objective of creating a portfolio is risk minimization.

*** Expected Rate of Return of portfolio.**

It is the weighted average of expected returns of assets included in the portfolio. i.e

$$E(R_P) = W_A \times E(R_A) + W_B \times E(R_B)$$

where,

$$W_A = \text{Weight of stock } A = \frac{\text{Amt. invested in Stock } A}{\text{Total Amount invested}}$$

$$W_B = \text{Weight of stock } B = \frac{\text{Amt. invested in stock } B}{\text{Total Amount invested}}$$

* Total Risk of portfolio (σ_p):

For two assets portfolio A and B

$$\sigma_p = \sqrt{W_A^2 \cdot \sigma_A^2 + W_B^2 \cdot \sigma_B^2 + 2 \cdot W_A \cdot W_B \cdot \underbrace{COV_{AB}}_{\rho_{AB} \times \sigma_A \times \sigma_B}}$$

For three assets portfolio A, B and C.

$$\sigma_p = \sqrt{W_A^2 \cdot \sigma_A^2 + W_B^2 \cdot \sigma_B^2 + W_C^2 \cdot \sigma_C^2 + 2 \cdot W_A \cdot W_B \cdot COV_{AB} + 2 \cdot W_B \cdot W_C \cdot COV_{BC} + 2 \cdot W_A \cdot W_C \cdot COV_{AC}}$$

* Calculation of covariance (COV.) and correlation (ρ)

Historical Data:

$$COV_{AB} = \frac{\Sigma [R_A - \bar{R_A}][R_B - E(R_B)]}{n-1}$$

Data with probability:

$$COV_{AB} = \Sigma P_j [R_A - E(R_A)][R_B - E(R_B)]$$

$$\text{Correlation } (\rho_{AB}) = \frac{COV_{AB}}{\sigma_A \times \sigma_B}$$

Problem 5.1

Sol^n

Given:

Investment in stock A = Rs. 30,000 ⎤
Investment in stock B = Rs. 70,000 ⎦ = Rs. 100,000

Expected return on stock A, $E(R_A) = 10\%$

Expected return on stock B, $E(R_B) = 15\%$

Expected return on portfolio, $E(R_p) = ?$

We know that,

$$E(R_p) = W_A \times E(R_A) + W_B \times E(R_B)$$

$$= 0.30 \times 10\% + 0.70 \times 15\%$$

$$= 13.5\%$$

where,

Weight of stock A, $(W_A) = \frac{30{,}000}{100{,}000} = 0.30$

Weight of stock B, $(W_B) = \frac{70{,}000}{100{,}000} = 0.70$

∴ The expected return on the portfolio is 13.5%.

Problem 5.2

Soln

a. Calculation of Average return over the four-year period.

Year	Beginning value	Ending value	$HPR = \frac{EV - BV}{BV} \times 100$
2012	Rs. 50,000	Rs. 55,000	10%
2013	55,000	58,000	5.45
2014	58,000	65,000	12.07
2015	65,000	70,000	7.69
			$\Sigma HPR = 35.2$

~~b. Calcn~~

Average Return of portfolio, $E(R_p) = \frac{\Sigma HPR}{n} = \frac{35.2}{4}$

$= 8.80\%$

b. Calculation of Standard Deviation of portfolio:

Year	R_p	$R_p - E(R_p)$	$[R_p - E(R_p)]^2$
2012	10%	1.20	1.44
2013	5.45	-3.35	11.22
2014	12.07	3.27	10.69
2015	7.69	-1.11	1.23
			$\Sigma [R_p - E(R_p)]^2 = 24.58$

$\therefore$ standard deviation, $\sigma_p = \sqrt{\frac{\Sigma [R_p - E(R_p)]^2}{n-1}} = \sqrt{\frac{24.58}{4-1}} = 2.86\%$

Problem 5.3

Solⁿ

a. Calculation of Expected Return:

Scenario	Prob.(P_j)	R_A	R_B	$P_j \times R_A$	$P_j \times R_B$
Recission	0.30	10%	50%	3%	15%
Normal economy	0.40	20	30	8	12
Boom	0.30	30	10	9	3
				$\Sigma P_j \times R_A = 20\%$	$\Sigma P_j \times R_B = 30\%$

∴ Expected Return of stock A, $E(R_A) = \Sigma P_j \times R_A = 20\%$

Expected Return of stock B, $E(R_B) = \Sigma P_j \times R_B = 30\%$

Weight of stock A, $(W_A) = 50\% = 0.50$

Weight of stock B, $(W_B) = 50\% = 0.50$

Expected return on portfolio, $E(R_p) = W_A \times E(R_A) + W_B \times E(R_B)$

$= 0.50 \times 20\% + 0.50 \times 30\%$

$= 25\%$

b. Calculation of Standard Deviation:

Scenario	Prob.(P_j)	$R_A - E(R_A)$	$R_B - E(R_B)$	$P_j[R_A - E(R_A)]^2$	$P_j[R_B - E(R_B)]^2$
Recission	0.30	−10	20	30	120
Normal	0.40	0	0	0	0
Boom	0.30	10	−20	30	120
				$\Sigma P_j[R_A - E(R_A)]^2 = 60$	$\Sigma P_j[R_B - E(R_B)]^2 = 240$

∴ standard deviation of stock A, $(\sigma_A) = \sqrt{\Sigma P_j [R_A - E(R_A)]^2}$

$= \sqrt{60} = 7.75\%$

∴ standard deviation of stock B, $(\sigma_B) = \sqrt{\Sigma P_j [R_B - E(R_B)]^2}$

$= \sqrt{240} = 15.49\%$

standard deviation of portfolio

$$\sigma_P = \sqrt{W_A^2 \cdot \sigma_A^2 + W_B^2 \cdot \sigma_B^2 + 2 \cdot W_A \cdot W_B \cdot COV_{AB}}$$

$$= \sqrt{(0.50)^2 \times (7.75)^2 + (0.50)^2 \times (15.49)^2 + 2 \times 0.50 \times 0.50 \times (-120)}$$

$= 3.87\%$

where,

Calculation of covariance (COV_{AB})

Scenario	P_j	$R_A - E(R_A)$	$R_B - E(R_B)$	$P_j [R_A - E(R_A)] [R_B - E(R_B)]$
Recession	0.30	−10	20	−60
Normal	0.40	0	0	0
Boom	0.30	10	−20	−60
				$COV_{AB} = -120$

Problem 5.4

Soln

Given:

	KWSC (A)	KEC (B)
Expected Return	14 %	16%
Standard deviation	5 %	9%

a. Weight of KWSC $(W_A) = 30\%$

Weight of KEC $(W_B) = 70\%$

Expected Return on portfolio, $E(R_p) = W_A \times E(R_A) + W_B \times E(R_B)$

$= 0.30 \times 14\% + 0.70 \times 16\%$

$= 15.4\%$

b. Calculation of standard deviation:

i. If a perfect positive correlation $(\rho_{AB}) = +1$

$$\sigma_p = \sqrt{W_A^2 \cdot \sigma_A^2 + W_B^2 \cdot \sigma_B^2 + 2 \cdot W_A \cdot W_B \cdot \rho_{AB} \cdot \sigma_A \cdot \sigma_B}$$

$$= \sqrt{(0.30)^2 \times (5)^2 + (0.70)^2 \times (9)^2 + 2 \times 0.30 \times 0.70 \times 1 \times 5 \times 9}$$

$= 7.80\%$

ii. If a slightly negative correlation $(\rho_{AB}) = -0.2$

$$\sigma_p = \sqrt{(0.30)^2 \times (5)^2 + (0.70)^2 \times (9)^2 + 2 \times 0.30 \times 0.70 \times (-0.2) \times 5 \times 9}$$

$= 6.18\%$

Problem 5.7

Soln

Given:

a. If stock M and stock N are perfectly positively correlated $(\rho_{MN}) = +1$

Weight of stock M, $(W_M) = 50\%$.
Weight of stock N, $(W_N) = 50\%$.

Expected Return on portfolio, $E(R_p) = W_M \times E(R_M) + W_N \times E(R_N)$
$= 0.50 \times 8\% + 0.50 \times 13\%$.
$= 10.5\%$

Standard deviation of portfolio

$$\sigma_p = \sqrt{W_M^2 \cdot \sigma_M^2 + W_N^2 \cdot \sigma_N^2 + 2 \cdot W_M \cdot W_N \cdot \rho_{MN} \cdot \sigma_M \cdot \sigma_N}$$
$$= \sqrt{(0.50)^2 \times (5\%)^2 + (0.50)^2 \times (10)^2 + 2 \times 0.50 \times 0.50 \times 1 \times 5 \times 10}$$
$$= 7.5\%$$

b. If stock M and stock N are uncorrelated $(\rho_{MN} = 0)$

Weight of stock M, $(W_M) = 50\%$
Weight of stock N, $(W_N) = 50\%$.

Expected Return on portfolio, $E(R_p) = W_M \times E(R_M) + W_N \times E(R_N)$
$= 0.50 \times 8\% + 0.50 \times 13\%$.
$= 10.5\%$

$$\sigma_p = \sqrt{W_M^2 \cdot \sigma_M^2 + W_N^2 \cdot \sigma_N^2 + 2 \cdot W_M \cdot W_N \cdot \rho_{MN} \cdot \sigma_M \cdot \sigma_N}$$

$$= \sqrt{(0.50)^2 \times (5)^2 + (0.50)^2 \times (10)^2 + 2 \times 0.50 \times 0.50 \times 0 \times 5 \times 10}$$

$= 5.59\ \%$

c. If stock M and stock N are perfectly negatively correlated ($\rho_{MN} = -1$)

Weight of stock M, (W_M) = 50%

Weight of stock N, (W_N) = 50 %

Expected Return on portfolio, $E(R_p) = W_M \times E(R_M) + W_N \times E(R_N)$

$= 0.50 \times 8\% + 0.50 \times 13\%$

$= 10.5\ \%$

Standard deviation of portfolio:

$$\sigma_p = \sqrt{W_M^2 \cdot \sigma_M^2 + W_N^2 \cdot \sigma_N^2 + 2 \cdot W_M \cdot W_N \cdot \rho_{MN} \cdot \sigma_M \cdot \sigma_N}$$

$$= \sqrt{(0.50)^2 \times (5)^2 + (0.50)^2 \times (10)^2 + 2 \times 0.50 \times 0.50 \times (-1) \times 5 \times 10}$$

$= 2.5\ \%$

Problem 5.8

Soln

Given:

Covariance between stock X and stock Y, (COV_{XY}) = 800

Correlation between stock X and stock Z, (ρ_{XZ}) = -0.5

a. Weight of security X $(W_x) = 33\%$
Weight of security Y $(W_Y) = 67\%$.

$$E(R_p) = W_X \cdot E(R_X) + W_Y \cdot E(R_Y)$$
$$= 0.33 \times 5\% + 0.67 \times 15\%$$
$$= 11.7\%$$

$$\sigma_p = \sqrt{W_X^2 \cdot \sigma_X^2 + W_Y^2 \cdot \sigma_Y^2 + 2 \times W_X \cdot W_Y \cdot COV_{xy}}$$
$$= \sqrt{0.33^2 \times 20^2 + 0.67^2 \times 40^2 + 2 \times 0.30 \times 0.67 \times 800}$$
$$= 33.4\%$$

b. Weight of security X $(W_X) = 33\%$
Weight of security Z $(W_Z) = 66\%$.

$$E(R_p) = W_X \cdot E(R_X) + W_Z \cdot E(R_Z)$$
$$= 0.33 \times 5\% + 0.67 \times 15\%$$
$$= 11.7\%$$

$$\sigma_p = \sqrt{W_X^2 \cdot \sigma_X^2 + W_Z^2 \cdot \sigma_Z^2 + 2 \times W_X \cdot W_Z \cdot \rho_{XZ} \cdot \sigma_X \cdot \sigma_Z}$$
$$= \sqrt{0.33^2 \times 20^2 + 0.67^2 \times 40^2 + 2 \times 0.33 \times 0.67 \times (-0.50) \times 20 \times 40}$$
$$= 24.19\%$$

c. Portfolio XZ has lower standard deviation because correlation beetween security X and Z is negative. Diversification is better with portfolio XZ.

Problem 5.5

Sol^n

a. Calculation of Expected Return

Year	R_X	R_Y	R_Z	$R_{XY} = 0.50 \times R_X + 0.50 \times R_Y$	$R_{XZ} = 0.50 \times R_X + 0.50 \times R_Z$
2012	16%	17%	14%	16.5%	15%
2013	17	16	15	16.5%	16
2014	18	15	16	16.5	17
2015	19	14	17	16.5	18
	$\Sigma R_X = 70$			$\Sigma R_{XY} = 66$	$\Sigma R_{XZ} = 66$

∴ Expected Return of assets X, $E(R_X) = \frac{\Sigma R_X}{n} = \frac{70}{4} = 17.5\%$

∴ Expected Return of portfolio XY, $E(R_{XY}) = \frac{\Sigma R_{XY}}{n} = \frac{66}{4} = 16.5\%$

∴ Expected Return of portfolio XZ, $E(R_{XZ}) = \frac{\Sigma R_{XZ}}{n} = \frac{66}{4} = 16.5\%$

b. Calculation of standard Deviation:

Year	R_X	R_{XY}	R_{XZ}	$[R_X - E(R_X)]^2$	$[R_{XY} - E(R_{XY})]^2$	$[R_{XZ} - E(R_{XZ})]^2$
2012	16%	16.5%	15%	2.25	0	2.25
2013	17	16.5	16	0.25	0	0.25
2014	18	16.5	17	0.25	0	0.25
2015	19	16.5	18	2.25	0	2.25
				$\Sigma[R_X - E(R_X)]^2 = 5$	$\Sigma[R_{XY} - E(R_{XY})]^2 = 0$	$\Sigma[R_{XZ} - E(R_{XZ})]^2 = 5$

∴ Standard deviation of stock X

$$\sigma_X = \sqrt{\frac{\Sigma[R_X - E(R_X)]^2}{n-1}} = \sqrt{\frac{5}{4-1}} = 1.29\%$$

∴ Standard deviation of portfolio XY

$$\sigma_{XY} = \sqrt{\frac{\Sigma[R_{XY} - E(R_{XY})]^2}{n-1}} = \sqrt{\frac{0}{4-1}} = 0$$

∴ Standard deviation of portfolio XZ

$$\sigma_{XZ} = \sqrt{\frac{\Sigma[R_{XZ} - E(R_{XZ})]^2}{n-1}} = \sqrt{\frac{5}{4-1}} = 1.29\%$$

c. On the basis of findings in part (a) and (b) we would recommend portfolio XY due to zero risk.

Problem 5.6

Soln

a. Calculation of Expected Return

Year	R_A	R_B	R_C
2016	12%	16%	12%
2017	14	14	14
2018	16	12	16
	$\Sigma R_A = 42$	$\Sigma R_B = 42$	$\Sigma R_C = 42$

$\therefore$ Expected Return of stock A, $E(R_A) = \frac{\Sigma R_A}{n} = \frac{42}{3} = 14\%$

$\therefore$ Expected Return of stock B, $E(R_B) = \frac{\Sigma R_B}{n} = \frac{42}{3} = 14\%$

$\therefore$ Expected Return of stock C, $E(R_C) = \frac{\Sigma R_C}{n} = \frac{42}{3} = 14\%$

b. Calculation of standard deviation:

Year	$[R_A - E(R_A)]^2$	$[R_B - E(R_B)]^2$	$[R_C - E(R_C)]^2$
2016	4	4	4
2017	0	0	0
2018	4	4	4
	$\Sigma[R_A - E(R_A)]^2 = 8$	$\Sigma[R_B - E(R_B)]^2 = 8$	$\Sigma[R_C - E(R_C)]^2 = 8$

$\therefore$ Standard deviation of Assets A, $\sigma_A = \sqrt{\frac{\Sigma[R_A - E(R_A)]^2}{n-1}} = \sqrt{\frac{8}{3-1}} = 2\%$

$\therefore$ Standard deviation of Assets B, $\sigma_B = \sqrt{\frac{\Sigma[R_B - E(R_B)]^2}{n-1}} = \sqrt{\frac{8}{3-1}} = 2\%$

$\therefore$ Standard deviation of Assets C, $\sigma_C = \sqrt{\frac{\Sigma[R_C - E(R_C)]^2}{n-1}} = \sqrt{\frac{8}{3-1}} = 2\%$

c. Calculation of expected return of each portfolio

Year	R_A	R_B	R_C	$R_{AB} = 0.5 \times R_A + 0.5 \times R_B$	$R_{AC} = 0.5 \times R_A + 0.5 \times R_C$
2016	12%	16%	12%	14	12%
2017	14	14	14	14	14
2018	16	12	16	14	16
				$\Sigma R_{AB} = 42$	$\Sigma R_{AC} = 42$

∴ Expected Return of portfolio AB, $E(R_{AB}) = \frac{\Sigma R_{AB}}{n} = \frac{42}{3} = 14\%$

∴ Expected Return of portfolio AC, $E(R_{AC}) = \frac{\Sigma R_{AC}}{n} = \frac{42}{3} = 14\%$

d. Assets A and assets B are negatively correlated since their returns are moving in opposite direction but Assets A and assets C are perfectly positively correlated because their returns are moving in same direction.

e. Calculation of standard Deviation of each portfolio:

year	R_{AB}	R_{AC}	$R_{AB} - E(R_{AB})$	$R_{AC} - E(R_{AC})$	$[R_{AB} - E(R_{AB})]^2$	$[R_{AC} - E(R_{AC})]^2$
2016	14%	12%	0	-2	0	4
2017	14	14	0	0	0	0
2018	14	16	0	2	0	4
					$\Sigma[R_A - E(R_A)]^2 = 0$	$\Sigma[R_{AC} - E(R_{AC})]^2 = 8$

∴ Standard deviation of portfolio AB

$$\sigma_{AB} = \sqrt{\frac{\Sigma[R_{AB} - E(R_{AB})]^2}{n-1}} = \sqrt{\frac{0}{3-1}} = 0$$

∴ standard deviation of portfolio AC

$$\sigma_{AC} = \sqrt{\frac{\Sigma[R_{AC} - E(R_{AC})]^2}{n-1}} = \sqrt{\frac{8}{3-1}} = 2\%$$

f. We would Recommend portfolio AB due to zero Risk.

g. Calⁿ of Expected Return and standard deviation of portfolio ABC.

Year	R_A	R_B	R_C	R_{ABC}	$R_{ABC} - E(R_{ABC})$	$[R_{ABC} - E(R_{ABC})]^2$
2016	12%	16%	12%	13.33%	-0.67	0.4489
2017	14	14	14	14	0	0
2018	16	12	16	14.67	0.67	0.4489
				ΣR_{ABC} = 42%		$\Sigma[R_{ABC} - E(R_{ABC})]^2 = 0.8978$

∴ Expected Return of portfolio ABC, $E(R_{ABC}) = \frac{\Sigma R_{ABC}}{n} = \frac{42}{3} = 14\%$

∴ Standard Deviation of portfolio ABC, $\sigma_{ABC} = \sqrt{\frac{\Sigma[R_{ABC} - E(R_{ABC})]^2}{n-1}}$

$$= \sqrt{\frac{0.8978}{3-1}} = 0.67\%$$

forming portfolio of Assets A, B and C the expected returns remains constant but the risk has been reduced than portfolio AC.

Problem 5.9

Soln

a. Calculation of expected return and standard deviation:

Quarter	R_1	R_2	$R_1-E(R_1)$	$R_2-E(R_2)$	$[R_1-E(R_1)]^2$	$[R_2-E(R_2)]^2$
1	7.2%	0.6%	3.55	-0.613	12.6025	0.3758
2	12.9	6.0	9.25	4.787	85.5625	22.915
3	7.4	0.6	3.75	-0.613	14.0625	0.3758
4	-2.1	9.7	-5.75	8.487	33.0625	72.029
5	4.0	-18.0	0.35	-19.213	0.1225	369.139
6	6.9	6.1	3.25	4.887	10.5625	23.883
7	-8.4	-1.8	-12.05	-3.013	145.2025	9.078
8	1.3	6.5	-2.35	5.287	5.5225	27.952
	$\Sigma R_1 = 29.2$	$\Sigma R_2 = 9.7$			$\Sigma[R_1-E(R_1)]^2 = 306.7$	$\Sigma[R_2-E(R_2)]^2 = 525.75$

∴ Expected Return of stock 1, $E(R_1) = \frac{\Sigma R_1}{n} = \frac{29.2}{8} = 3.65\%$

∴ Expected Return of stock 2, $E(R_2) = \frac{\Sigma R_2}{n} = \frac{9.7}{8} = 1.21\%$

∴ Standard Deviation of stock 1, $\sigma_1 = \sqrt{\frac{\Sigma[R_1-E(R_1)]^2}{n-1}} = \sqrt{\frac{306.7}{8-1}} = 6.62\%$

∴ Standard Deviation of stock 2, $\sigma_2 = \sqrt{\frac{\Sigma[R_2-E(R_2)]^2}{n-1}} = \sqrt{\frac{525.75}{8-1}} = 8.67\%$

b. Calculation of correlation coefficient.

Quarter	$R_1 - E(R_1)$	$R_2 - E(R_2)$	$[R_1 - E(R_1)][R_2 - E(R_2)]$
1	3.55	-0.613	-2.17615
2	9.25	4.787	44.27975
3	3.75	-0.613	-2.29875
4	-5.75	8.487	-48.80025
5	0.35	-19.213	-6.72455
6	3.25	4.887	15.88275
7	-12.05	-3.013	36.30665
8	-2.35	5.287	-12.42445
			$\Sigma[R_1 - E(R_1)][R_2 - E(R_2)] = 24.04$

Covariance between stock 1 and stock 2

$$COV_{12} = \frac{\Sigma[R_1 - E(R_1)][R_2 - E(R_2)]}{n-1}$$

$$= \frac{24.04}{8-1} = 3.43$$

Correlation coefficient between stock 1 and stock 2

$$\rho_{12} = \frac{COV_{12}}{\sigma_1 \times \sigma_2} = \frac{3.43}{6.62 \times 8.67} = 0.06$$

c. Weight of stock 1, $(W_1) = 50\%$
Weight of stock 2, $(W_2) = 50\%$

Expected Return on portfolio, $E(R_p) = W_1 \times E(R_1) + W_2 \times E(R_2)$
$= 0.50 \times 3.65\% + 0.50 \times 1.21\%$
$= 2.43\%$

Standard deviation of portfolio

$$\sigma_p = \sqrt{W_1^2 \cdot \sigma_1^2 + W_2^2 \cdot \sigma_2^2 + 2 \cdot W_1 \cdot W_2 \cdot COV_{12}}$$

$$= \sqrt{(0.50)^2 \times (6.62)^2 + (0.50)^2 \times (8.67)^2 + 2 \times 0.50 \times 0.50 \times 3.43}$$

$= 5.61\%$

The portfolio Risk is lower than individual risk of stock 1 and stock 2.

d. Return on risk free assets $(R_f) = 2\%$
Weight of risk free assets $(W_{RF}) = 10\%$
Weight of portfolio $(W_p) = 90\%$

Expected Return, $E(R_p) = W_{RF} \times R_f + W_p \times R_p$
$= 0.10 \times 2\% + 0.90 \times 2.43$
$= 2.39\%$

Risk of portfolio, $\sigma_p = W_p \times \sigma_p = 0.90 \times 5.61\% = 5.05\%$

Problem 5.10

Soln

a. Calculation of expected rate of return:

Prob. (P_j)	Return on market (R_m)	Return on stock J (R_j)	$P_j \times R_m$	$P_j \times R_j$
0.3	15%	20%	4.5%	6%
0.4	9	5	3.6	2
0.3	18	12	5.4	3.6
			$\Sigma P_j \times R_m = 13.5$	$\Sigma P_j \times R_j = 11.6$

∴ Expected Return of ~~sto~~ market, $E(R_m) = \Sigma P_j \times R_m = 13.5\%$

∴ Expected Return of stock j, $E(R_j) = \Sigma P_j \times R_j = 11.6\%$

b. Calculation of standard deviation:

Prob. (P_j)	$R_m - E(R_m)$	$R_j - E(R_j)$	$P_j[R_m - E(R_m)]^2$	$P_j[R_j - E(R_j)]^2$
0.3	1.5	8.4	0.675	21.168
0.4	-4.5	-6.6	8.1	17.424
0.3	4.5	0.4	6.075	0.048
			$\Sigma P_j[R_m - E(R_m)]^2 = 14.85$	$\Sigma P_j[R_j - E(R_j)]^2 = 38.64$

∴ standard deviation of market, $\sigma_m = \sqrt{\Sigma P_j[R_m - E(R_M)]^2} = \sqrt{14.85} = 3.85\%$

∴ standard deviation of stock j. $\sigma_j = \sqrt{\Sigma P_j[R_j - E(R_j)]^2} = \sqrt{38.64} = 6.22\%$

c.

Beta coefficient of stock j $(\beta_j) = \frac{COV_{jm}}{\sigma_m^2}$

$= \frac{16.2}{(3.85)^2}$

$= 1.091$

where,

Calculation of covariance:

Prob. (P_j)	$R_m - E(R_m)$	$R_j - E(R_j)$	$P_j[R_m - E(R_m)][R_j - E(R_j)]$
0.3	1.5	8.4	3.78
0.4	-4.5	-6.6	11.88
0.3	4.5	0.4	0.54

$\Sigma P_j[R_m - E(R_m)][R_j - E(R_j)] = COV_{jm}$
$= 16.2$

The beta coefficient of stock j is greater than the market beta 1, which indicates that stock j is risky than the market and it is said to be an aggressive stock.

* Total Risk of Investment / Investment Risk:

Risk can be defined as probability of happening some unfavourable event in the future. Higher the deviations (difference) among investment returns higher will be the total risk of investment and vice-versa.

Standard Deviation is used to measure the total risk of an investment and it is denoted by sigma(σ).

* Division of Total Risk

Total Risk is measured by standard deviation can be divided into following two types:

1. Diversifiable Risk / Unsystematic Risk / Assets specific Risk
2. Non-Diversifiable Risk / Systematic Risk / Market Risk

1. Unsystematic Risk:

It is that portion of total risk which can be diversified away through the creation of portfolio. As this portion of total risk can be diversified away, investors should not require any risk premium (additional return due to risk) for bearing this portion of total risk.

2. Systematic Risk

It is that portion of total risk which cannot be diversified away even through the creation of portfolio. As this portion of total risk cannot be diversified away. Investors should require risk premium for bearing this portion of total risk. This risk is represented by Beta coefficient(β).

Total Risk of Asset j = σ_j

Systematic Risk of Asset j [SR_j]

$$= \frac{COV_{jm}}{\sigma_m}$$

$$= \frac{\rho_{jm} \times \sigma_j \times \cancel{\sigma_m}}{\cancel{\sigma_m}}$$

$$= \rho_{jm} \times \sigma_j$$

Unsystematic Risk of Assets j [UR_j]

$$= \sigma_j - SR_j$$

$$= \sigma_j - \frac{COV_{jm}}{\sigma_m}$$

$$= \sigma_j - \rho_{jm} \times \sigma_j$$

$$= \sigma_j (1 - \rho_{jm})$$

* Beta Coefficient of Asset j [B_j] = $\frac{\text{Systematic Risk of Assets j}}{\sigma_m}$

$$= \frac{\frac{COV_{jm}}{\sigma_m}}{\sigma_m}$$

$$= \frac{COV_{jm}}{\sigma_m^2}$$

$$= \frac{\rho_{jm} \times \sigma_j \times \cancel{\sigma_m}}{\sigma_m \times \cancel{\sigma_m}}$$

$$= \frac{\rho_{jm} \times \sigma_j}{\sigma_m}$$

* Interpretation of Beta coefficient (β):

1. If $\beta_j = 1$ ⇒ This beta is called average beta and the assets having average beta is called Average assets.

2. If $\beta_j > 1$ ⇒ This beta is called aggresive beta and the assets having aggressive beta is called aggresive assets.

3. If $\beta_j < 1$ ⇒ This beta is called defensive beta and the assets having defensive beta is called defensive assets.

* Required Rate of Return [R]:

It is the minimum rate of return required by an investor to invest in the given investment opportunity. It is calculated by using following formula:

$$R = R_f + (R_m - R_f)\beta_j$$

where,

R_f = Risk free Rate (Treasury)

R_m = Return on market

$(R_m - R_f)$ = Market Risk premium

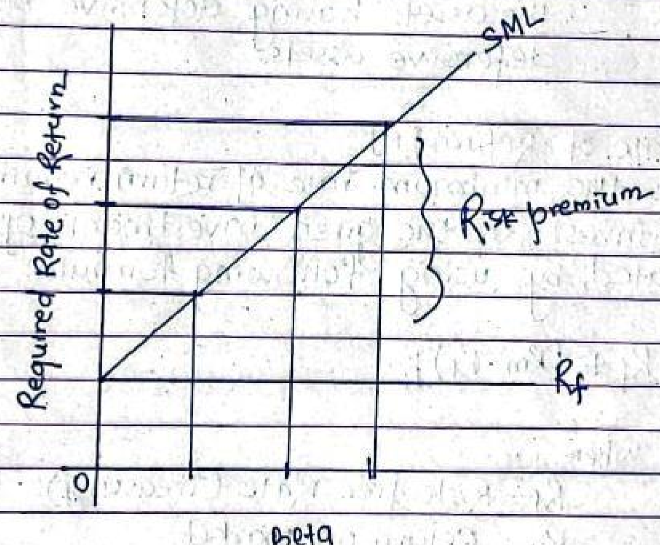
β_j = Beta of Asset j.

Decision:

1. If Expected Return > Required Return → Buy or purchase
2. If Expected Return < Required Return → Do not purchase
3. If Expected Return = Required Return → Indifferent

* Security Market Line (SML):

It is a graph which shows the relationship between Beta coefficient and Required rate of return of assets.



Problem 5.11

Soln

a. Calculation of average Return:

Year	R_m	R_A	R_B
2006	6%	11%	16%
2007	2	8	11
2008	-13	-4	-10
2009	-4	3	3
2010	-8	0	-3
2011	16	19	30
2012	10	14	22
2013	15	18	29
2014	8	12	19
2015	13	17	26
	$\Sigma R_m = 45\%$	$\Sigma R_A = 98\%$	$\Sigma R_B = 143\%$

∴ Average Return on market, $E(R_m) = \frac{\Sigma R_m}{n} = \frac{45}{10} = 4.5\%$

∴ Average Return on stock A, $E(R_A) = \frac{\Sigma R_A}{n} = \frac{98}{10} = 9.8\%$

∴ Average Return on stock B, $E(R_B) = \frac{\Sigma R_B}{n} = \frac{143}{10} = 14.3\%$

b. Calculation of variance and standard deviation:

Year	$R_m - E(R_m)$	$R_A - E(R_A)$	$R_B - E(R_B)$	$[R_m - E(R_m)]^2$	$[R_A - E(R_A)]^2$	$[R_B - E(R_B)]^2$
2006	1.5	1.2	1.7	2.25	1.44	2.89
2007	-2.5	-1.8	-3.3	6.25	3.24	10.89
2008	-17.5	-13.8	-24.3	306.25	190.44	590.49
2009	-8.5	-6.8	-11.3	72.25	46.24	127.69
2010	-12.5	-9.8	-17.3	156.25	96.04	299.29
2011	11.5	9.2	15.7	132.25	86.64	246.49
2012	5.5	4.2	7.7	30.25	17.64	59.29
2013	10.5	8.2	14.7	110.25	67.24	216.09
2014	3.5	2.2	4.7	12.25	4.84	22.09
2015	8.5	7.2	11.7	72.25	51.84	136.89
				$\Sigma[R_m - E(R_m)]^2 = 900.5$	$\Sigma[R_A - E(R_A)]^2 = 563.60$	$\Sigma[R_B - E(R_B)]^2 = 1712.10$

∴ Variance of market, $\sigma_m^2 = \dfrac{\Sigma[R_m - E(R_m)]^2}{n-1} = \dfrac{900.5}{10-1} = 100.06$

∴ Standard deviation of market, $\sigma_m = \sqrt{100.06} = 10\%$

∴ Variance of investment A, $\sigma_A^2 = \dfrac{\Sigma[R_A - E(R_A)]^2}{n-1} = \dfrac{563.60}{10-1} = 62.62$

∴ Standard deviation of investment A, $\sigma_A = \sqrt{62.62} = 7.91\%$

∴ Variance of Investment B, $\sigma_B^2 = \dfrac{\Sigma[R_B - E(R_B)]^2}{n-1} = \dfrac{1712.10}{10-1} = 190.23$

$\therefore$ Standard deviation of stock B, $\sigma_B = \sqrt{190.23} = 13.79\%$

c. Calculation of covariance (COV_{AB})

Year	$[R_M - E(R_m)][R_A - E(R_A)]$	$[R_m - E(R_m)][R_B - E(R_B)]$
2006	1.80	2.55
2007	4.50	8.25
2008	241.50	425.25
2009	57.80	96.05
2010	122.50	216.25
2011	105.80	180.55
2012	23.10	42.35
2013	86.10	154.35
2014	7.70	16.45
2015	61.20	99.45
	$\Sigma[R_m - E(R_m)][R_A - E(R_A)] = 712$	$\Sigma[R_m - E(R_m)][R_B - E(R_B)] = 1241.5$

Covariance between market return and investment A

$$COV_{AM} = \frac{\Sigma[R_m - E(R_m)][R_A - E(R_A)]}{n-1} = \frac{712}{10-1} = 79.1$$

Covariance between market return and investment B

$$COV_{Bm} = \frac{\Sigma[R_m - E(R_m)][R_B - E(R_B)]}{n-1} = \frac{1241.5}{10-1} = 137.9$$

d. Calculation of correlation:

Correlation between market and Investment A

$$\rho_{AM} = \frac{COV_{AM}}{\sigma_A \times \sigma_m} = \frac{79.1}{7.91 \times 10} = 1$$

Correlation between market and Investment B

$$\rho_{BM} = \frac{COV_{BM}}{\sigma_B \times \sigma_m} = \frac{137.9}{13.79 \times 10} = 1$$

e. Calculation of Beta coefficient:

Beta coefficient of investment A, $(\beta_A) = \frac{COV_{Am}}{\sigma_m^2} = \frac{79.1}{100.06} = 0.79$

Beta coefficient of Investment B, $(\beta_B) = \frac{COV_{BM}}{\sigma_m^2} = \frac{137.9}{100.06} = 1.38$

f. Since, Investment A has Beta coefficient less than 1, it is less risky than the market whereas investment B has beta coefficent more than 1, so it is more risky than the market.

Problem 5.12

Sol^n

Calculation of Required Rate of Return using CAPM

Security	Risk free Rate (R_f)	Market Return (R_m)	Beta (β)	Required Rate of Return $R = R_f + (R_m - R_f)\beta$
A	5%	8%	1.30	$= 5\% + (8\% - 5\%)1.30 = 8.9\%$
B	8	13	0.90	$= 8\% + (13\% - 8\%)0.90 = 12.5\%$
C	9	12	−0.20	$= 9\% + (12\% - 9\%)(-0.20) = 8.4\%$
D	10	15	1.00	$= 10\% + (15\% - 10\%)1.00 = 15\%$
E	6	10	0.60	$= 6\% + (10\% - 6\%)0.60 = 8.4\%$

Problem 5.13

Sol^n

Given:

Risk free Rate (R_f) = 3%

Beta (β) = 1.25

Expected Return, E(R) = 14%

Market Return (R_m) = 13%

Required Rate of Return (R) $= R_f + (R_m - R_f)\beta$

$= 3\% + (13\% - 3\%)1.25$

$= 15.5\%$

Jagar should leave his funds in the T-bill because its expected return (14%) is lower than required rate of return (15.5%).

Problem 5.14

Soln

Given:

Risk free Rate (R_f) = 6%

Return on market (R_m) = 14%

Beta of Company A (β_A) = 1.55

Beta of Company B (β_B) = 0.75

Price of stock A = Rs. 38

Price of stock B = Rs. 23 → Total = 38+23 = Rs. 61

a. Required Rate of Return for common stock A

$$E(R_A) = R_f + (R_m - R_f)\beta_A$$
$$= 6\% + (14\% - 6\%)\ 1.55$$
$$= 18.4\%$$

b. Required Rate of Return for common stock B

$$E(R_B) = R_f + (R_m - R_f)\beta_B$$
$$= 6\% + (14\% - 6\%)\ 0.75$$
$$= 12\%$$

c. Weight of stock A (W_A) = $\frac{38}{61}$ = 0.63

Weight of stock B (W_B) = $\frac{23}{61}$ = 0.37

Required Rate of Return for portfolio = $W_A \times E(R_A) + W_B \times E(R_B)$

$$= 0.63 \times 18.4\% + 0.37 \times 12\%$$
$$= 16.03\%$$

OR,

$$\beta_p = W_A \times \beta_A + W_B \times \beta_B$$
$$= 0.63 \times 1.55 + 0.37 \times 0.75$$
$$= 1.254$$

$$E(R_p) = R_f + (R_m - R_f)\beta_p$$
$$= 6\% + (14\% - 6\%) 1.254$$
$$= 16.03\%$$

Problem 5.15

Soln

Given:

- Risk free rate (R_f) = 6%
- Expected Return on market portfolio, $E(R_m)$ = 11%
- Standard Deviation of the market portfolio, (σ_m) = 11%
- Correlation Coefficient of Asset A and market (ρ_{AM}) = +0.80
- Standard deviation of the return of Asset A (σ_A) = 9%

a. Beta coefficient for Asset A

$$\beta_A = \frac{\rho_{Am} \times \sigma_A \times \sigma_m}{\sigma_m^2}$$

$$= \frac{0.80 \times 9 \times 11}{11 \times 11}$$

$$= 0.65$$

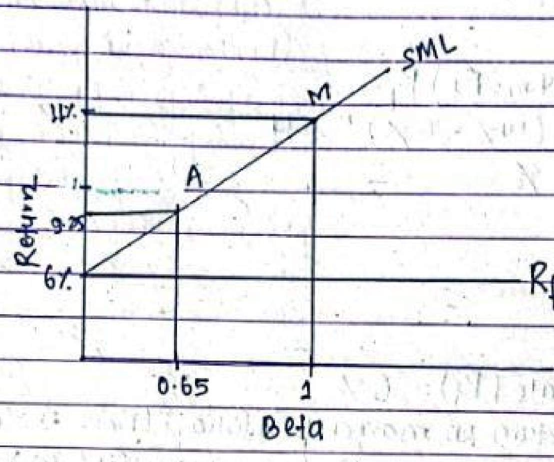
Since, Beta of Asset A is less than 1, it is a defensive asset.

b. Expected Return for Asset A, $E(R_A) = R_f + (R_m - R_f)\beta_A$

$= 6\% + (11\% - 6\%)0.65$

$= 9.25\%$

c.



d. Systematic Risk for Asset A $= \frac{COV_{AM}}{\sigma_m} = \frac{\rho_{AM} \times \sigma_A \times \sigma_m}{\sigma_m}$

$= \frac{0.80 \times 9 \times 11}{11}$

$= 7.2\%$

Unsystematic Risk $= \sigma_A(1 - \rho_{AM})$

$= 9\%(1 - 0.80) = 1.8\%$

Total Risk of asset A is 9% where 7.2% is systematic and 1.8% is unsystematic.

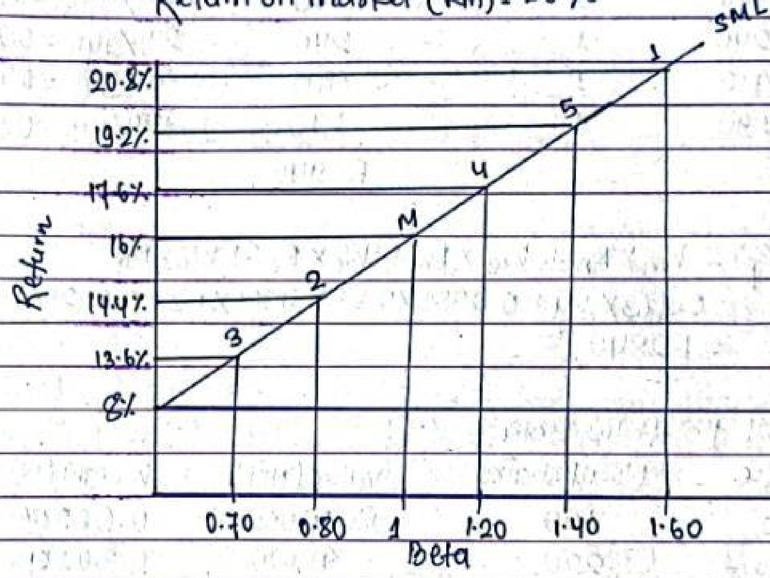
Problem 5.16

Sol^n

Given:

Risk free Rate (R_f) = 8%

Return on market (R_m) = 16%



Cal^n of Required Rate of Return:

$$R = R_f + (R_m - R_f)\beta$$

R_1 = 8% + (16% − 8%) 1.60 = 20.8%

R_2 = 8% + (16% − 8%) 0.80 = 14.4%

R_3 = 8% + (16% − 8%) 0.70 = 13.6%

R_4 = 8% + (16% − 8%) 1.20 = 17.6%

R_5 = 8% + (16% − 8%) 1.40 = 19.2%

Problem 5.17

Soln

a. Calculation of portfolio Beta:

Stock	Price	No. of shares	Value(NXP)	Weight(W)
A	Rs.100	1	100	100/940 = 0.1063
B	240	1	240	240/940 = 0.2553
C	410	1	410	410/940 = 0.4361
D	190	1	190	190/940 = 0.2021
			Rs.940	

$$\text{Portfolio Beta}(\beta_P) = W_A \times \beta_A + W_B \times \beta_B + W_C \times \beta_C + W_D \times \beta_D$$
$$= 0.1063 \times 1.4 + 0.2553 \times 0.8 + 0.4361 \times 1.3 + 0.2021 \times 1.8$$
$$= 1.2840$$

b. Calculation of portfolio Beta:

Stock	Price	No. of shares	value(NXP)	Weight(W)
A	Rs.100	100	Rs.10,000	0.0629
B	240	200	48,000	0.3019
C	410	200	82,000	0.5157
D	190	100	19,000	0.1195
			159,000	

$$\text{Portfolio Beta}(\beta_P) = W_A \times \beta_A + W_B \times \beta_B + W_C \times \beta_C + W_D \times \beta_D$$
$$= 0.0629 \times 1.4 + 0.3019 \times 0.8 + 0.5157 \times 1.3 + 0.1195 \times 1.8$$
$$= 1.2151$$

c. Calculation of Required rate of return on each stock:

$$E(R) = R_f + (R_m - R_f)\beta$$

$$E(R_A) = 8\% + (12\% - 8\%)\,1.2 = 12.8\%$$

$$E(R_B) = 8\% + (12\% - 8\%)\,0.8 = 11.2\%$$

$$E(R_C) = 8\% + (12\% - 8\%)\,1.5 = 14\%$$

$$E(R_D) = 8\% + (12\% - 8\%)\,2 = 16\%$$

Problem 5.18

Soln

a. Ranking of stocks from most risky to least risky

Stock	Beta	Rank	Remarks
A	0.80	2	
B	1.40	1	most risky
C	-0.30	3	least risky

b. If market increases by 12%, the change in stock's return is given by:

Stock	Beta(β)	Increase in market return(ΔR_m)	$\beta \times \Delta R_m$
A	0.80	12%	9.6%
B	1.40	12%	16.8%
C	- 0.30	12%	- 3.6%

c. If the market return declines by 5%.

Stock	Beta (β)	ΔRm	β×ΔRm
A	0.8	-5%	-4%
B	1.4	-5%	-7%
C	-0.3	-5%	+1.5%

d. If we felt the stock market was about to experience a significant decline, we would be most likely to add to stock C because it has negative beta means that when market declines it increases.

e. If we anticipated a major stock market increases, we would be most likly to add stock B to our portfolio because its beta is the highest and highest beta results highest return.

Problem 5.19

Soln

Given:

$\beta_1 = 1.2$ $\beta_2 = 0.9$

$W_1 = 50\%$ $W_2 = 50\%$

a. Beta of portfolio $(\beta_p) = W_1 \times \beta_1 + W_2 \times \beta_2$

$= 0.50 \times 1.2 + 0.50 \times 0.9$

$= 1.05$

b. Portfolio Beta (β_p) = 1.1 , W_1 = ? , W_2 = ?

We have,

$$\boxed{\begin{aligned} W_1 + W_2 &= 1 \\ W_2 &= 1 - W_1 \end{aligned}}$$

$$\beta_p = W_1 \times \beta_1 + W_2 \times \beta_2$$

$$1.1 = W_1 \times 1.2 + (1-W_1)\,0.9$$

$$1.1 = 1.2\,W_1 + 0.9 - 0.9\,W_1$$

$$1.1 - 0.9 = 0.3\,W_1$$

$$W_1 = \frac{0.2}{0.3} = 0.6667 \text{ or } 66.67\%$$

$$W_2 = 1 - W_1 = 1 - 0.6667 = 0.3333 \text{ or } 33.33\%$$

Problem 5.20

Soln

a. Calculation of Beta for Portfolio A and B

Asset	Asset Beta	W_A	W_B	Beta x W_A	Beta x W_B
1	1.3	0.10	0.30	0.13	0.39
2	0.7	0.30	0.10	0.21	0.07
3	1.25	0.10	0.20	0.125	0.25
4	1.1	0.10	0.20	0.11	0.22
5	0.9	0.40	0.20	0.36	0.18
				β_A = 0.935	β_B = 1.11

b. Portfolio A has beta less than 1, so it is less risky than market whereas Portfolio B has beta more than 1, so it is more risky than market. Since, beta of stock B is higher than the beta of stock A. stock B portfolio is more risky.

c.

Risk free Rate (R_f) = 2%

Market Return (R_m) = 12%

Required Rate of Return, $E(R) = R_f + (R_m - R_f) \times \beta$

Portfolio A, $E(R_A) = 2\% + (12\% - 2\%) \times 0.935 = 11.35\%$

Portfolio B, $E(R_B) = 2\% + (12\% - 2\%) \times 1.11 = 13.1\%$

d. Calculation of Expected Return of portfolio A and B.

Assets	R_j	W_A	W_B	$W_A \times R_j$	$W_B \times R_j$
1	16.5%	0.10	0.30	1.65%	4.95%
2	12%	0.30	0.10	3.6	1.2
3	15%	0.10	0.20	1.5	3
4	13%	0.10	0.20	1.3	2.6
5	7%	0.40	0.20	2.8	1.4
				$E(R_A) = 10.85\%$	$E(R_B) = 13.15\%$

We would choose portfolio B because its average return (13.15%) is greater than the required return (13.1%).

Problem 5.21

Soln

a. Calculation of Expected Return

State of Economy	Probability (P_j)	R_{MF}	R_{CS}	R_{CD}	$P_j \times R_{MF}$	$P_j \times R_{CS}$	$P_j \times R_{CD}$
Weak growth	0.3333	8%	6%	7%	2.6664	1.9998	2.3331
Moderate	0.3333	10	12	7	3.333	3.9996	2.3331
Strong	0.3333	12	15	7	3.9996	4.9995	2.3331
					= 10%	= 11%	= 7%

∴ The expected return on Mutual fund, $E(R_{MF}) = \Sigma P_j \times R_{MF} = 10\%$

∴ The expected return on Common Stock, $E(R_{CS}) = \Sigma P_j \times R_{CS} = 11\%$

∴ The expected return on certificate of Deposit, $E(R_{CD}) = \Sigma P_j \times R_{CD} = 7\%$

Hence, Common stock provides highest expected Return.

b. Calculation of Standard Deviation:

State of Economy	P_j	$R_{MF} - E(R_{MF})$	$R_{CS} - E(R_{CS})$	$R_{CD} - E(R_{CD})$	$P_j [R_{MF} - E(R_{MF})]^2$
Weak growth	0.3333	-2	-5	0	1.3332
Moderate	0.3333	0	1	0	0
Strong	0.3333	2	4	0	1.3332
					= 2.6664

$P_j [R_{CS} - E(R_{CS})]^2$	$P_j [R_{CD} - E(R_{CD})]^2$
8.3325	0
0.3333	0
5.3328	0
= 13.9986	= 0

∴ Standard deviation of Mutual Fund $(\sigma_{MF}) = \sqrt{\sum P_j [R_{MF} - E(R_{MF})]^2}$

$= \sqrt{2.6664} = 1.6329\%$

∴ Standard deviation of Common stock $(\sigma_{CS}) = \sqrt{\sum P_j [R_{CS} - E(R_{CS})]^2}$

$= \sqrt{13.9986} = 3.7415\%$

∴ Standard deviation of certificate of Deposit $(\sigma_{CD}) = \sqrt{\sum P_j [R_{CD} - E(R_{CD})]^2}$

$= \sqrt{0} = 0$

Certificate of Deposit is least risky in terms of standard deviation because it has zero risk.

Common stock is the most risky in terms of beta because it has higher beta than mutual fund and certificate of deposit.

c. Portfolio 'X' | Portfolio Y

Weight of Mutual fund $(W_{MF}) = 75\%$. Weight of Common stock $(W_{CS}) = 50\%$.
Weight of Common stock $(W_{CS}) = 25\%$. Wight of certificate of Deposit $(W_{CD}) = 50\%$.

Standard Deviation of portfolio X

$$\sigma_X = \sqrt{W_{MF}^2 \cdot \sigma_{MF}^2 + W_{CS}^2 \cdot \sigma_{CS}^2 + 2 \cdot W_{MF} \cdot W_{CS} \cdot COV_{MF,CS}}$$

$$= \sqrt{(0.75)^2 \times (1.6329)^2 + (0.25)^2 \times (3.7415)^2 + 2 \times 0.75 \times 0.25 \times 6}$$

$$= 2.150\%$$

Working Note:

Calculation of covariance between MF and CS ($COV_{MF,CS}$)

State	$R_{MF} - E(R_{MF})$	$R_{CS} - E(R_{CS})$	P_j	$P_j [R_{MF} - E(R_{MF})][R_{CS} - E(R_{CS})]$
Weak	-2	-5	0.3333	3.333
Moderate	0	1	0.3333	0
Strong	2	4	0.3333	2.666
				6%

∴ Covariance between MF and CS ($COV_{MF,CS}$) $= \sum P_j [R_{MF} - E(R_{MF})](R_{CS} - E(R_{CS})]$

$$= 6\%$$

Standard deviation of portfolio Y

$$\sigma_Y = \sqrt{W_{CS}^2 \times \sigma_{CS}^2 + W_{CD}^2 \times \sigma_{CD}^2 + 2 \cdot W_{CS} \cdot W_{CD} \cdot COV_{CS,CD}}$$

$$= \sqrt{(0.50)^2 \times (3.7415)^2 + (0.50)^2 \times (0)^2 + 2 \times 0.50 \times 0.50 \times 0}$$

$$= 1.870\%$$

Portfolio Y is less ricky in terms of standard deviation due to lower standard deviation.

Beta of portfolio X

$$\beta_X = W_{MF} \times \beta_{MF} + W_{CS} \times \beta_{CS}$$
$$= 0.75 \times 1 + 0.25 \times 1.2$$
$$= 1.05$$

Beta of portfolio Y

$$\beta_Y = W_{CS} \times \beta_{CS} + W_{CD} \times \beta_{CD}$$
$$= 0.50 \times 1.2 + 0.50 \times 0$$
$$= 0.6$$

Portfolio Y is least risky in terms of beta due to lower beta.

d. The standard deviation measures the total risk. Total risk has two components: systematic risk and unsystematic risk. Beta is a measure of systematic risk. In a well diversified portfolio only the systematic risk is relevant because of that we calculate beta.