Bond Valuation

Chapter-9
BOND VALUATION

Bond:

- It is a legal written promisory note.
- It is a agreement between issuers and investors.
- It is a long term debt whose life is more than a year.
- Maturity period is fixed. (Generally 5 Yrs, 10 Yrs, 20 Yrs, 30 Yrs).
- Interest is a income received from bond.
- Bond can be called before its maturity period which is also known as callable bond.

Terms:

1. Maturity value (M) / Par value / Face value
2. Term to Maturity (n) / Maturity period
3. Coupon rate (c) / interest rate (i)
4. Required rate of return (K_d) / Yield to Maturity (YTM) / Market rate / discount rate.
5. Call period (n_c) for callable bond
6. Call price (CP) (par value + premium)

* Types of Bond:

Bond

- Redeemable Bond (whose life is fixed)
  - Coupon Bond ($V = I_{1-n} + M$)
  - Zero coupon Bond ($V = M$)
- Irredemable Bond (whose life is not fixed) or Perpetual Bond or Consol Bond ($V = I_{1-\infty}$)

* Valuation of Bond:

- If interest is paid annually:

1. Coupon Bond:

$$\text{Intrinsic Value of Bond } (V_0) = I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$$

OR,

$$V_0 = I \left[\frac{1 - \frac{1}{(1+K_d)^n}}{K_d} \right] + \frac{M}{(1+K_d)^n}$$

2. Zero Coupon Bond:

$$\text{Intrinsic Value of Bond } (V_0) = M \times PVIF_{kd\%,n}$$

OR,

$$V_0 = \frac{M}{(1+K_d)^n}$$

3. Perpetual Bond:

Intrinsic Value of Bond (V_0) = I/K_d

• If interest is paid semi-annually:

Changes:

$I \longrightarrow I/2$

$K_d \longrightarrow K_d/2$

$n \longrightarrow n \times 2$

* Relationship between coupon rate (C), Required rate of return (K_d/YTM) and Value of Bond (V_0).

If,	$C = K_d$	V_0 = Rs. 1000 (Par)
If,	$C > K_d$	V_0 = more than 1000
If,	$C < K_d$	V_0 = less than 1000

Conclusion:

· Coupon rate and value of bond has direct relation i.e increase in coupon rate increases the value of bond and vice-versa.

· Required Rate of Return or Yield to Maturity and value of bond has inverse relation i.e increase in YTM decreases the value of bond and vice versa.

- Higher the maturity period higher will be the value of bond and vice-versa.

Decision:

1. If Market price (given) = Intrinsic value (calculated)
   ⇒ Correctly valued (Equilibrium) ⇒ Indifference

2. If Market price (given) < Intrinsic value (calculated)
   ⇒ Undervalued → Buy/Purchase

3. If Market price (given) > Intrinsic value (calculated)
   ⇒ Overvalued → Not purchase (sell)

* Bond Yields:

1. Rate of Return (HPR) $= \frac{\text{Ending price} - \text{Beginning price} + \text{Interest}}{\text{Beginning price}} \times 100$

$$= \frac{\text{Selling price} - \text{Purchase price} + \text{Interest}}{\text{Purchase price}} \times 100$$

$\boxed{P_0 = V_0}$

$$= \frac{P_1 - P_0 + I_1}{P_0} \times 100$$

$$= \frac{P_1 - P_0}{P_0} \times 100 + \frac{I}{P_0} \times 100$$

HPR / YTM = Capital Gain Yield + Current Yield

2. Coupon Yield = $\frac{\text{Interest}}{\text{Face value}} \times 100$

3. Current Yield = $\frac{\text{Interest}}{\text{Current Market price}} \times 100$

$= \frac{I}{V_0} \times 100$

* Yield To Maturity (YTM):

Yield to Maturity is the annualized rate of return that an investor can realize if the bond is held until the maturity.

Assumptions of YTM:

1. The bond should be held until the maturity.
2. The bond should pay all promised cash flows.
3. The bond should not be called by the issuer before maturity.
4. All intermediates cash flows in terms of coupon interest from bonds are assumed to reinvest at the rate of return equal to YTM.

Formula:

a. Perpetual Bond:

$V_0 = I/K_d \rightarrow YTM$

$\boxed{\therefore YTM = I/V_0}$

b. Zero coupon Bond:

$$V_0 = \frac{M}{(1+K_d)^n}$$

$K_d \rightarrow$ YTM

$$(1+YTM)^n = \frac{M}{V_0}$$

$$\therefore \boxed{YTM = \left[\frac{M}{V_0}\right]^{1/n} - 1}$$

c. Coupon Bond

- If interest is paid annually

Step-1: $\text{Approximate YTM} = \dfrac{I + \dfrac{M - V_0}{n}}{\dfrac{M + 2V_0}{3}} \times 100$

Step-2: Try at low rate

$$TPV_{LR} = I \times PVIFA_{LR\%,n} + M \times PVIF_{LR\%,n}$$

(Ans = V_0 থেকে বেশী)

Step-3: Try at High rate

$$TPV_{HR} = I \times PVIFA_{HR\%,n} + M \times PVIF_{HR\%,n}$$

(Ans = V_0 থেকে কম)

Step-4 : By Interpolation:

$$\text{Annual YTM} = LR + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

- If interest is paid semi-annually

Changes:

$YTM \longrightarrow YTM/2$

$I \longrightarrow I/2$

$n \longrightarrow n \times 2$

Step-5 : Nominal YTM = Semi-annual YTM × 2

Step-6 : Effective Annual YTM = $(1 + \text{Semi-annual YTM})^2 - 1$

* Yield To Call (YTC) :

An Yield to call is the annualized rate of return that bondholders will realize if the bond is called at the stated date before the maturity.

Assumptions of YTC :

1. The given bond must be hold until its call period.
2. The issuer should not default to pay the promised amount at promised date.
3. The coupons from the given bond must be reinvested at the rate of YTC of the given bond.

Formula: Same as YTM

Changes

YTM $\rightarrow$ YTC

$n \rightarrow n_c$ (call Period)

$M \rightarrow CP$ (call price)

* Duration:

It can be defined as average time required to recover the incomes from the given bond or it can simply be defined as average life or maturity of the bond. It is propounded by Frederick Macaulay so it is called Macaulay Duration.

Note:

1. The duration of Coupon Bond is always less than its life.
2. The duration of Zero Coupon Bond is always equals to its life.

Formula:

$$\text{Duration (D)} = \frac{1+Y}{Y} - \frac{(1+Y) + n(C-Y)}{C[(1+Y)^n - 1] + Y}$$

$$\text{Modified Duration (MD)} = \frac{D}{1+Y}$$

$$\text{Change in price } (\Delta P) = -1 \times MD \times \Delta I$$

Where,

Y = Yield To Maturity
C = Coupon rate
n = number of payments
MD = Modified Duration
ΔI = Change in Interest Rate
100 Basis points = 1 %

In case of semi-annual payment

Changes.

$Y \rightarrow Y/2$
$C \rightarrow C/2$
$n \rightarrow 2n$

Annual Duration = $D/2$

* Why duration is calculated?

It indicates the weighted average number of years that cash flows occur from the bond. Duration is a better measure of time structure of bonds than years to maturity because duration reflects the amount and timing of every cash flow rather than merely the length of time until the final payment occurs. Moreover, duration is a measure of bond's interest rate risk. So duration is calculated.

Problem 9.1

Soln

Given:

a. Perpetual Bond:

Par value (M) = Rs. 1000

Coupon rate (C) = 7 %

Interest Amount (I) = M×C = 1000×0.07 = Rs. 70

Required rate of return (Kd) = 9 %

Value of Bond (V_0) = ?

We know that,

$$V_0 = \frac{I}{Kd} = \frac{70}{0.09} = Rs.\ 777.77$$

b. Zero coupon Bond:

Maturity period (n) = 10 Yrs

Par value (M) = Rs. 1000

Required rate of return (Kd) = 12 %

Value of bond (V_0) = ?

We know that,

$$V_0 = M \times PVIF_{Kd\%,n}$$
$$= 1000 \times PVIF_{12\%,10}$$
$$= 1000 \times 0.3220$$
$$= Rs.\ 322$$

OR,

$$V_0 = \frac{M}{(1+Kd)^n} = \frac{1000}{(1+0.12)^{10}} = Rs.\ 322$$

c. Coupon Bond:

Par value (M) = Rs. 1000

Maturity period (n) = 8 Yrs

Coupon rate (c) = 7.5 %

Interest Amount (I) = M X C = 1000 X 0.075 = Rs. 75

Yield To Maturity (Kd) = 12 %

Value of Bond (V_0) = ?

We know that,

$$V_0 = I \times PVIFA_{kd\%, n} + M \times PVIF_{kd\%, n}$$

$$= 75 \times PVIFA_{12\%, 8} + 1000 \times PVIF_{12\%, 8}$$

$$= 75 \times 4.9676 + 1000 \times 0.4039$$

$$= Rs.\ 776.47$$

OR,

$$V_0 = I \left[\frac{1 - \frac{1}{(1+Kd)^n}}{Kd} \right] + \frac{M}{(1+Kd)^n}$$

$$= 75 \left[\frac{1 - \frac{1}{(1+0.12)^8}}{0.12} \right] + \frac{1000}{(1+0.12)^8}$$

$$= Rs.\ 776.47$$

Problem 9.2

Sol^n

Given: Coupon Bond:

Face Value (M) = Rs. 1000

Coupon rate (c) = 8% (semi-annually)

Maturity period (n) = 5 years

Interest Amount (I) = M × C = 1000 × 0.08 = Rs. 80

(a) Required rate of return (Kd) = 6%

Value of Bond (V_0) = ?

We know that,

$$V_0 = \frac{I}{2} \times PVIFA_{\frac{kd}{2}\%,\, n\times 2} + M \times PVIF_{\frac{kd}{2}\%,\, n\times 2}$$

$$= \frac{80}{2} \times PVIFA_{\frac{6}{2}\%,\, 5\times 2} + 1000 \times PVIF_{\frac{6}{2}\%,\, 5\times 2}$$

$$= 40 \times PVIFA_{3\%,10} + 1000 \times PVIF_{3\%,10}$$

$$= 40 \times 8.5302 + 1000 \times 0.7441$$

$$= Rs.\ 1085.31$$

(b) Required rate of return (Kd) = 8%

Value of Bond (V_0) = ?

$$V_0 = \frac{I}{2} \times PVIFA_{\frac{kd}{2}\%,\, n\times 2} + M \times PVIF_{\frac{kd}{2}\%,\, n\times 2}$$

$$= \frac{80}{2} \times PVIFA_{\frac{8}{2}\%,\, 5\times 2} + 1000 \times PVIF_{\frac{8}{2}\%,\, 5\times 2}$$

$$= 40 \times PVIFA_{4\%,10} + 1000 \times PVIF_{4\%,10}$$

$= 40 \times 8.1109 + 1000 \times 0.6756$

$= Rs.\ 1000$

c. Required rate of return (Kd) = 10%

$V_0 = \frac{I}{2} \times PVIFA_{\frac{kd}{2}\%, n\times 2} + M \times PVIF_{\frac{kd}{2}\%, n\times 2}$

$= \frac{80}{2} \times PVIFA_{\frac{10}{2}\%, 5\times 2} + 1000 \times PVIF_{\frac{10}{2}\%, 5\times 2}$

$= 40 \times PVIFA_{5\%,10} + 1000 \times PVIF_{5\%,10}$

$= 40 \times 7.7217 + 1000 \times 0.6139$

$= Rs.\ 922.78$

Problem 9.3

Soln

Given: Coupon Bond:

Coupon Rate (c) = 8.5 % , $I = 1000 \times 0.085 = Rs.85$

Face Value (M) = Rs. 1000

Maturity period (n) = 5 years

Required rate of Return (kd) = 12%

Value of Bond (V_0) = ?

$V_0 = I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$

$= 85 \times PVIFA_{12\%,5} + 1000 \times PVIF_{12\%,5}$

$= 85 \times 3.6048 + 1000 \times 0.5674$

$= Rs.\ 873.81$

If interest is paid semi-annually:

$$V_0 = \frac{I}{2} \times PVIFA_{\frac{kd}{2}\%, n\times2} + M \times PVIF_{\frac{kd}{2}\%, n\times2}$$

$$= \frac{85}{2} \times PVIFA_{\frac{12}{2}\%, 5\times2} + 1000 \times PVIF_{\frac{12}{2}\%, 5\times2}$$

$$= 42.5 \times PVIFA_{6\%, 10} + 1000 \times PVIF_{6\%, 10}$$

$$= 42.5 \times 7.3601 + 1000 \times 0.5584$$

$$= Rs.\ 871.2$$

Problem 9.4

Soln

Given: Coupon

Interest (I) = Rs. 90

Par value (M) = Rs. 1000

Maturity period (n) = 20 years

Required rate of return (Kd) = 8%

a. Value of Bond $(V_0) = I \times PVIFA_{kd\%, n} + M \times PVIF_{kd\%, n}$

$$= 90 \times PVIFA_{8\%, 20} + 1000 \times PVIF_{8\%, 20}$$

$$= 90 \times 9.8181 + 1000 \times 0.2145$$

$$= Rs.\ 1098.13$$

b. Required rate of return (Kd) = 10%

ii. Required rate of return (K_d) = 6%

Problem 9.5

Soln

Given: Coupon

	Bond A	Bond B
Annual Interest (I)	Rs. 80	Rs. 80
Maturity value (M)	Rs. 1000	Rs. 1000
Maturity period (n)	15 years	1 year

a. i. Required Rate of Return (K_d) = 4%

Bond A

$V_0 = I \times PVIFA_{kd\%, n} + M \times PVIF_{kd\%, n}$

$= 80 \times PVIFA_{4\%, 15} + 1000 \times PVIF_{4\%, 15}$

$= 80 \times 11.1184 + 1000 \times 0.5553$

$= Rs.\ 1444.78$

Bond B

$V_0 = I \times PVIFA_{kd\%, n} + M \times PVIF_{kd\%, n}$

=

=

ii. Required Rate of Return (Kd) = 8%.

Bond A

Bond B

iii. Required Rate of Return (Kd) = 12%

Bond A

Bond B

b. The value of longer term bond fluctuates more in compare to shorter term bond because, the longer term bonds are more exposed to maturity risk or price risk.

Problem 9.6

Soln

Given: Coupon Bond:

Annual coupon Rate (c) = 6.5 %

Interest Amount (I) = M X C = 1000 X 0.065 = Rs. 65

Maturity period (n) = 10 yrs

Yield to Maturity (YTM/Kd) = 7 %

a. The bondholders will receive interest of Rs. 65 each year.

b. $$\text{Current Yield} = \frac{I}{V_0} \times 100$$

$$= \frac{65}{964.83} \times 100$$

$$= 6.74\%$$

Working Note:

$$V_0 = I \times PVIFA_{kd\%, n} + M \times PVIF_{kd\%, n}$$

$$= 65 \times PVIFA_{7\%, 10} + 1000 \times PVIF_{7\%, 10}$$

$$= 65 \times 7.0236 + 1000 \times 0.5083$$

$$= Rs.\ 964.83$$

b. The bond sells at Rs. 964.83

c. Yield To Maturity (YTM|Kd) = 6 %

$$\text{Value of Bond}(V_0) = I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$$
$$= 65 \times PVIFA_{6\%,10} + 1000 \times PVIF_{6\%,10}$$
$$= 65 \times 7.3601 + 1000 \times 0.5584$$
$$= Rs.\ 1036.8$$

d. The coupon rate and value of bond has positive relation i.e increase in coupon rate increases the value of bond and vice-versa.
The yield to maturity and value of bond has inverse relation i.e increase in yield to maturity decreases the value of bond and vice-versa.

Problem 9.7

Soln

Given: Coupon Bond

Coupon Interest (I) = Rs. 90
Par value (M) = Rs. 1000
Maturity period (n) = 10 years
Required rate of return (Kd) = 10%

$$\text{Value of Bond today}(V_0) = I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$$
$$= 90 \times PVIFA_{10\%,10} + 1000 \times PVIF_{10\%,10}$$
$$= 90 \times 6.1446 + 1000 \times 0.3855$$
$$= Rs.\ 938.514$$

After 2 Years:

Remaining life (n) = 10 - 2 = 8 Yrs

Value of Bond (V_2) = $I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$

$= 90 \times PVIFA_{10\%,8} + 1000 \times PVIF_{10\%,8}$

$= 90 \times 5.3349 + 1000 \times 0.4665$

= Rs. 946.64

After 6 years:

Remaining life (n) = 10 - 6 = 4 Yrs

Value of bond (V_6) = $I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$

$= 90 \times PVIFA_{10\%,4} + 1000 \times PVIF_{10\%,4}$

$= 90 \times 3.1699 + 1000 \times 0.6830$

= Rs. 968.29

Problem 9.8

Soln

Given: Coupon Bond:

Coupon rate (c) = 8 %

Maturity period (n) = 18 Yrs

Yield (Kd) = 10 %

After a Year

Yield (Kd) = 9%

Remaining life (n) = 18 - 1 = 17 Yrs

Interest (I) = M X C = 1000 X 0.08

= Rs. 80

Value of bond today (V_0) = $I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$

$= 80 \times PVIFA_{10\%,18} + 1000 \times PVIF_{10\%,18}$

$= 80 \times 8.2014 + 1000 \times 0.1799$

$=$ Rs. 836.01

Value of bond in 1 year (V_1) = $I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$

$= 80 \times PVIFA_{9\%,17} + 1000 \times PVIF_{9\%,17}$

$= 80 \times 8.5436 + 1000 \times 0.2311$

$=$ Rs. 914.59

Holding Period Return (HPR) = $\frac{V_1 - V_0 + I_1}{V_0} \times 100$

$= \frac{914.59 - 836.01 + 80}{836.01} \times 100$

$= 18.97\%$

Problem 9.9

Soln

Given: Coupon Bond

Coupon rate (C) = 10%

Maturity period (n) = 25 Yrs

Selling price (V_0) = Rs. 1200

Interest (I) = $M \times C = 1000 \times 0.10 =$ Rs. 100

Current Yield (CY) = $\frac{I}{V_0} \times 100 = \frac{100}{1200} \times 100 = 8.33\%$

Calculation of Yield To Maturity (YTM):

- If Interest is paid annually

Step-1: Approximat YTM = $\dfrac{I + \dfrac{M-V_0}{n}}{\dfrac{M+2V_0}{3}} \times 100$

$= \dfrac{100 + \dfrac{1000-1200}{25}}{\dfrac{1000+2\times1200}{3}} \times 100.$

$= 8.12\%.$

Step-2: Try at low rate i.e 8%

$TPV_{LR} = I \times PVIFA_{YTM\%,n} + M \times PVIF_{YTM\%,n}$
$= 100 \times PVIFA_{8\%,25} + 1000 \times PVIF_{8\%,25}$
$= 100 \times 10.6748 + 1000 \times 0.1460$
$=$ Rs. 1213.48

Step-3: Try at High rate i.e 9%

$TPV_{HR} = I \times PVIFA_{YTM\%,n} + M \times PVIF_{YTM\%,n}$
$= 100 \times PVIFA_{9\%,25} + 1000 \times PVIF_{9\%,25}$
$= 100 \times 9.8226 + 1000 \times 0.1160$
$=$ Rs. 1098.26

Step-4: By interpolation:

$$\text{Annual YTM} = LR + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

$$= 8\% + \frac{1213.48 - 1200}{1213.48 - 1098.26} \times (9\% - 8\%)$$

$$= 8.12\%$$

• If interest is paid semi-annually:

Step-1: $$\text{Approximate } \frac{YTM}{2} = \frac{\frac{I}{2} + \frac{M - V_0}{n \times 2}}{\frac{M + 2V_0}{3}} \times 100$$

$$= \frac{\frac{100}{2} + \frac{1000 - 1200}{25 \times 2}}{\frac{1000 + 2 \times 1200}{3}} \times 100$$

$$= 4.05\%$$

Step-2: Try at low rate i.e 4%

$$TPV_{LR} = \frac{I}{2} \times PVIFA_{\frac{YTM}{2}\%, n\times 2} + M \times PVIF_{\frac{YTM}{2}\%, n \times 2}$$

$$= \frac{100}{2} \times PVIFA_{4\%, 25\times 2} + 1000 \times PVIF_{4\%, 25\times 2}$$

$$= 50 \times PVIFA_{4\%, 50} + 1000 \times PVIF_{4\%, 50}$$

$= 50 \times 21.4822 + 1000 \times 0.1407$

$= Rs.\ 1214.81$

Step-3: Try at High Rate i.e 5%

$$TPV_{HR} = \frac{I}{2} \times PVIFA_{\frac{YTM}{2}, n\times2} + M \times PVIF_{\frac{YTM}{2}, n\times2}$$

$$= \frac{100}{2} \times PVIFA_{5\%, 25\times2} + 1000 \times PVIF_{5\%, 25\times2}$$

$$= 50 \times PVIFA_{5\%, 50} + 1000 \times PVIF_{5\%, 50}$$

$$= 50 \times 18.2559 + 1000 \times 0.0872$$

$= Rs.\ 1000$

Step-4: By Interpolation:

$$\text{Semi-annual YTM} = LR\% + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

$$= 4\% + \frac{1214.81 - 1200}{1214.81 - 1000} \times (5\% - 4\%)$$

$= 4.07\%$

Step-5: Nominal | Annual YTM = Semi-annual YTM × 2

$= 4.07 \times 2 = 8.14\%$

Step-6: Effective Annual YTM $= (1 + \text{semi annual YTM})^2 - 1$

$= (1 + 0.0407)^2 - 1$

$= 8.31\%$

Problem 9.10

Sol^n

Given:

Coupon rate (C) = 12% (semi-annual payment)

Maturity period (n) = 10 Yrs

Maturity value (M) = Rs. 1000

Interest Amount (I) = M×C = 1000 × 0.12 = Rs. 120

Selling price (V_0) = 95% of 1000 = Rs. 950

$$\text{Current yield} = \frac{I}{V_0} \times 100 = \frac{120}{950} \times 100 = 12.63\,\%$$

Calculation of YTM:

Step-1: $$\text{Approximate } \frac{YTM}{2} = \frac{\frac{I}{2} + \frac{M - V_0}{n \times 2}}{\frac{M + 2V_0}{3}} \times 100$$

$$= \frac{\frac{120}{2} + \frac{1000 - 950}{10 \times 2}}{\frac{1000 + 2 \times 950}{3}} \times 100$$

$$= 6.47\,\%$$

Step-2: Try at low rate i.e 6%

$$TPV_{LR} = \frac{I}{2} \times PVIFA_{\frac{YTM}{2}\%,\, n\times 2} + M \times PVIF_{\frac{YTM}{2}\%,\, n\times 2}$$

$$= \frac{120}{2} \times PVIFA_{6\%, 10\times2} + 1000 \times PVIF_{6\%, 10\times2}$$

$$= 60 \times PVIFA_{6\%, 20} + 1000 \times PVIF_{6\%, 20}$$

$$= 60 \times 11.4699 + 1000 \times 0.3118$$

$$= Rs.\ 1000$$

Step-3: Try at High Rate i.e 7%

$$TPV_{HR} = \frac{I}{2} \times PVIFA_{\frac{YTM}{2}\%, n\times2} + M \times PVIF_{\frac{YTM}{2}\%, n\times2}$$

$$= \frac{120}{2} \times PVIFA_{7\%, 10\times2} + 1000 \times PVIF_{7\%, 10\times2}$$

$$= 60 \times PVIFA_{7\%, 20} + 1000 \times PVIF_{7\%, 20}$$

$$= 60 \times 10.5940 + 1000 \times 0.2584$$

$$= Rs.\ 894.04$$

Step-4: Semi-Annual $YTM = LR + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$

$$= 6\% + \frac{1000 - 950}{1000 - 894.04} \times (7\% - 6\%)$$

$$= 6.47\%$$

Step-5: Annual. YTM = Semi-annual YTM × 2

$$= 6.47\% \times 2 = 12.94\%$$

Step-6: Effective Annual $YTM = (1 + \text{semiannual } YTM)^2 - 1$

$$= (1 + 0.0647)^2 - 1$$

$$= 13.36\%$$

Problem 9.11

Soln

Given:

a. Market price (V_0) = Rs. 850

Maturity period (n) = 10 years

Maturity value (M) = Rs. 1000

Coupon rate (C) = 12%

Interest (I) = M×C = 1000×0.12 = Rs. 120

Yield to Maturity (YTM) = ?

Step-1: Approximate YTM = $\dfrac{I + \dfrac{M - V_0}{n}}{\dfrac{M + 2V_0}{3}} \times 100$

$= \dfrac{120 + \dfrac{1000 - 850}{10}}{\dfrac{1000 + 2 \times 850}{3}} \times 100$

= 15%

Step-2: Try at 15%

$TPV_{HR} = I \times PVIFA_{YTM\%,n} + M \times PVIF_{YTM\%,n}$

$= 120 \times PVIFA_{15\%,10} + 1000 \times PVIF_{15\%,10}$

$= 120 \times 5.0188 + 1000 \times 0.2472$

= Rs. 849.46

Step-3: Try at 14%

$TPV_{LR} = I \times PVIFA_{YTM\%,n} + M \times PVIF_{YTM\%,n}$

$= 120 \times PVIFA_{14\%,10} + 1000 \times PVIF_{14\%,10}$

$= 120 \times 5.2161 + 1000 \times 0.2697$

$=$ Rs. 895.63

Step-4: By Interpolation:

$$\text{Annual YTM} = LR + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

$$= 14\% + \frac{895.63 - 850}{895.63 - 849.46} \times (15\% - 14\%)$$

$$= 14.99\%$$

b. Zero Coupon Bond:

Selling price (V_0) = Rs. 321

Maturity value (M) = Rs. 1000

Maturity period (n) = 10 years

Yield to Maturity (YTM) = ?

$$YTM = \left[\frac{M}{V_0}\right]^{1/n} - 1$$

$$= \left[\frac{1000}{321}\right]^{1/10} - 1$$

$$= 12\%$$

c. Perpetual Bond:

Selling price (V_0) = Rs.800

Coupon rate (c) = 10%

Par value (M) = Rs.1000

Interest (I) = M×C = 1000×0.10 = Rs.100

Yield To Maturity (YTM) = ?

$$YTM = I/V_0 = \frac{100}{800} = 12.5\%$$

Problem 9.12

Soln

Given: Coupon Bond:

Maturity period (n) = 10 years

Selling price (V_0) = Rs. 985

Face value (M) = Rs. 1000

Coupon rate (c) = 7%

Interest Amount (I) = M×C = 1000×0.07 = Rs.70

a. $$\text{Current Yield} = \frac{I}{V_0} \times 100 = \frac{70}{985} \times 100 = 7.11\%$$

b. Yield To Maturity (YTM) = ?

Note: Solve yourself (step 1 - step 4)

Ans = 7.22%

c. Remaining life = 10 - 3 = 7 Yrs , YTM = 7.22 %

Value of bond (V_3) = ?

$V_3 = I \times PVIFA_{YTM\%, n} + M \times PVIF_{YTM\%, n}$

$= I \left[\dfrac{1 - \dfrac{1}{(1+YTM)^n}}{YTM} \right] + \dfrac{M}{(1+YTM)^n}$

$= 70 \left[\dfrac{1 - \dfrac{1}{(1+0.0722)^7}}{0.0722} \right] + \dfrac{1000}{(1+0.0722)^7}$

= Rs. 988.23

Problem 9.13

Soln

Given: Coupon:

Maturity period (n) = 9 Years

Coupon rate (c) = 8 %

Selling price (V_0) = Rs. 901.40

Interest (I) = M × C = 1000 × 0.08 = Rs. 80

a. Current Yield = $\dfrac{I}{V_0} \times 100 = \dfrac{80}{901.40} \times 100$ = 8.88 %

b. Yield To Maturity (YTM) = ?

Do Yourself (Step 1 - Step 4) Ans = 9.67 %

c. Capital Gain Yield = YTM − Current Yield
= 9.67 − 8.88
= 0.79 %

Problem 9.14

Soln

Given: Coupon:

Maturity period (n) = 5 Years

Face value (M) = Rs. 1000

Annual coupon rate (c) = 8%

Current yield (CY) = 8.21 %

Yield To Maturity (YTM) = ?

Interest (I) = M×C = 1000×0.08 = Rs. 80

$$\text{Current Yield} = \frac{I}{V_0} \times 100$$

$$\frac{8.21}{100} = \frac{80}{V_0}$$

$\therefore V_0$ = Rs. 974.42

YTM : Do yourself (Step 1 − Step 4)

The coupon rate and the value of bond have direct relation i.e increase in coupon rate increases the value of bond and vice-versa.

YTM and value of bond have inverse relation i.e increase in ~~value~~ YTM decreases the value of bond and vice-versa.

Problem 9.15

Soln

Given: Coupon Bond:

Face value (M) = Rs. 1000

Maturity period (n) = 10 Years

Coupon rate (c) = 11 %

Interest $(I) = M \times C = 1000 \times 0.11 =$ Rs. 110

Current price (V_0) = Rs. 1175

Call period (n_c) = 5 Years

Call price (CP) = 109 % of 1000 = Rs. 1090

a. Calculation of Yield To Maturity (YTM) = ?

Step-1: Approximate YTM = $\dfrac{I + \dfrac{M - V_0}{n}}{\dfrac{M + 2V_0}{3}} \times 100$

$$= \dfrac{110 + \dfrac{1000 - 1175}{10}}{\dfrac{1000 + 2 \times 1175}{3}} \times 100$$

$$= 8.28\,\%$$

Step-2: Try at low rate i.e 8%

$TPV_{LR} = I \times PVIFA_{YTM\%, n} + M \times PVIF_{YTM\%, n}$

$= 110 \times PVIFA_{8\%, 10} + 1000 \times PVIF_{8\%, 10}$

$= 110 \times 6.7101 + 1000 \times 0.4632$

= Rs. 1201.31

Step-3: Try at High rate i.e 9%

$TPV_{HR} = I \times PVIFA_{YTM\%, n} + M \times PVIF_{YTM\%, n}$

$= 110 \times PVIFA_{9\%, 10} + 1000 \times PVIF_{9\%, 10}$

$= 110 \times 6.4177 + 1000 \times 0.4224$

$=$ Rs. 1128.34

Step-4: By Interpolation:

$$\text{Annual YTM} = LR + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

$$= 8\% + \frac{1201.31 - 1175}{1201.31 - 1128.34} \times (9\% - 8\%)$$

$$= 8.36\%$$

b. Calculation of Yield To Call (YTC): (same as YTM → changes

YTM → YTC

$n \rightarrow n_c$

M → CP

Step-1: Approximate YTC =

$$\frac{I + \frac{CP - V_0}{n_c}}{\frac{CP + 2V_0}{3}} \times 100$$

$$= \frac{110 + \frac{1090 - 1175}{5}}{\frac{1090 + 2 \times 1175}{3}} \times 100$$

$$= 8.11\%$$

Step-2: Try at low rate i.e 8%

$TPV_{LR} = I \times PVIFA_{YTC\%, n_c} + CP \times PVIF_{YTC\%, n_c}$

$= 110 \times PVIFA_{8\%, 5} + 1090 \times PVIF_{8\%, 5}$

$= 110 \times 3.9927 + 1090 \times 0.6806$

$=$ Rs. 1181.05

Step-3: Try at High rate i.e 9%

$TPV_{HR} = I \times PVIFA_{YTC\%, n_c} + CP \times PVIF_{YTC\%, n_c}$

Step-4: By Interpolation:

$$\text{Annual YTC} = LR\% + \frac{TPV_{LR} - V_0}{TPV_{LR} - TPV_{HR}} \times (HR - LR)$$

$=$

$= 8.13\%$

C. Investor of this bond might expect to earn 8.13% i.e yield to call. Because yeald to call is less than yield to maturity and company always try to reduce cost of debt. Hence, bond yield is expected to be called in 5 years and investor will earn only 8.13% i.e YTC.

Problem 9.16

Soln

Given:

	Bond A	Bond B
Maturity period (n)	20 Yrs	20 Yrs
Coupon rate (c)	9 % (semi)	8% (annual)
Yield (Kd)	10.5 %	7.5 %
Interest (I)	Rs. 90	Rs. 80

Bond A:

$$V_0 = \frac{I}{2} \times \left[\frac{1 - \frac{1}{(1+Kd/2)^{n\times2}}}{Kd/2} \right] + \frac{M}{\left(1+\frac{Kd}{2}\right)^{n\times2}}$$

$$= \frac{90}{2} \left[\frac{1 - \frac{1}{\left(1+\frac{0.105}{2}\right)^{20\times2}}}{0.105/2} \right] + \frac{1000}{\left(1+\frac{0.105}{2}\right)^{20\times2}}$$

$$= 45 \times 746.43 + 129.156$$

$$= Rs.\ 876$$

$$\text{Current Yield} = \frac{I}{V_0} \times 100 = \frac{90}{876} \times 100 = 10.27\ \%$$

Bond B:

$$V_0 = I\left[\frac{1 - \frac{1}{(1+K_d)^n}}{K_d}\right] + \frac{M}{(1+K_d)^n}$$

$$= 80\left[\frac{1 - \frac{1}{(1+0.075)^{20}}}{0.075}\right] + \frac{1000}{(1+0.075)^{20}}$$

$$= 815.56 + 235.41$$
$$= Rs.\ 1050.97$$

$$\text{Current Yield} = \frac{I}{V_0} \times 100 = \frac{80}{1050.97} \times 100 = 7.61\%$$

b. Bond A has higher YTM.

Problem 9.17

Soln

Given: Zero Coupon Bond

Maturity period (n) = 25 Yrs

Selling price (V_0) = 11.625% of 1000 = Rs. 116.25

Maturity value (M) = Rs. 1000

$$\text{Current Yield} = \frac{I}{V_0} \times 100 = \frac{0}{116.25} \times 100 = 0$$

$$\text{Promised Yield (YTM)} = \left[\frac{M}{V_0}\right]^{1/n} - 1$$

$$= \left[\frac{1000}{116.25}\right]^{1/25} - 1$$

$$= 8.99\%$$

If Yield (Kd) = 12% (semi-annual)

$$\text{Value of bond } (V_0) = M \times PVIF_{\frac{Kd}{2}\%,\, n\times 2}$$

$$= 1000 \times PVIF_{\frac{12}{2}\%,\, 25\times 2}$$

$$= 1000 \times PVIF_{6\%, 50}$$

$$= 1000 \times 0.0543$$

$$= Rs.\ 54.3$$

Problem 9.18

soln

Given:

Current price (V_0) = Rs.800

coupon rate (c) = 8%

Interest (I) = M×C = 1000×0.08 = Rs.80

Holding period(n) = 3 Yrs

selling price in 3 Yrs (V_3) = Rs.950

Realized yield (YTM) = ?

Do Yourself: (step1 - step4) Changes → $M \rightarrow V_3$

Holding period Return (HPR) $= \frac{SP - PP + I}{PP} \times 100$

$= \frac{950 - 800 + 80 \times 9/12}{800} \times 100$

$= \frac{150 + 60}{800} \times 100$

$= 26.25\%$

Problem 9.19

Soln

Given: Coupon Bond

Maturity period (n) = 10.5 Yrs

YTM = 10 %

Current price (V_0) = Rs. 860

Coupon Rate (c) = ?

$$V_0 = I\left[\frac{1 - \frac{1}{(1+YTM)^n}}{YTM}\right] + \frac{M}{(1+YTM)^n}$$

$$860 = I\left[\frac{1 - \frac{1}{(1+0.10)^{10.5}}}{0.10}\right] + \frac{1000}{(1+0.10)^{10.5}}$$

$860 = I \times 6.3239 + 367.60$

$860 - 367.60 = I \times 6.3239$

$\therefore I = 77.86$

Now,

$$I = M \times C$$

$$77.86 = 1000 \times C$$

$$\therefore C = \frac{77.86}{1000} = 7.78\%$$

Problem 9.20

Soln

Bond A: (Coupon Bond)

Coupon rate (c) = 10% , I = Rs. 100

Maturity period (n) = 7 years

Market interest rate (kd) = 8%

Value of Bond (V_0) = ?

$$V_0 = I \times PVIFA_{kd\%,n} + M \times PVIF_{kd\%,n}$$

$$= 100 \times PVIFA_{8\%,7} + 1000 \times PVIF_{8\%,7}$$

$$= 100 \times 5.2064 + 1000 \times 0.5835$$

$$= Rs.\ 1104.14$$

Bond B:

Current price (V_0) = Rs. 887

Maturity (n) = 10 years

Market interest rate (kd) = 12%

Coupon rate (c) = ?

Do Yourself (see Q.N.19)

Bond I   Coupon Bond:

Price (V_0) = Rs. 920

Coupon Rate (c) = 12%

Interest Amt (I) = 1000 × 0.12 = Rs. 120

Maturity (n) = 10 Yrs

Market Interest Rate (YTM) = ?

Do Yourself (step 1 – step 4)

Bond D   Coupon:

Price (V_0) = Rs. 1060

Coupon Rate (c) = 8%

Interest Amount (I) = 1000 × 0.08 = Rs. 80

Market Interest Rate (Kd) = 7.2% (YTM)

Maturity (n) = ?

$$\text{Approximate YTM} = \frac{I + \frac{M - V_0}{n}}{\frac{M + 2V_0}{3}}$$

$$0.072 = \frac{80 + \frac{1000 - 1060}{n}}{\frac{1000 + 2 \times 1060}{3}}$$

n = 11 Yrs

Bond E Zero Coupon Bond

Price (V_0) = Rs. 500

Maturity (n) = 5 Yrs

Market Interest Rate (YTM) = ?

$$YTM = \left[\frac{M}{V_0}\right]^{1/n} - 1$$

$$= \left[\frac{1000}{500}\right]^{1/5} - 1$$

$$= 14.86\ \%$$

Problem 9.21

Soln

Given:

Selling price (V_0) = Rs. 1000

Maturity period (n) = 3 years

Coupon rate (c) = 7 %

Yield To Maturity (Y) = 7 % [$\because V_0 = 1000$]

a. Bond's Duration (D) = $\frac{1+Y}{Y} - \frac{(1+Y)+n(c-Y)}{c[(1+Y)^n - 1]+Y}$

$$= \frac{1+0.07}{0.07} - \frac{(1+0.07)+3(0.07-0.07)}{0.07[(1+0.07)^3-1]+0.07}$$

$$= 2.81 \text{ years}$$

b. Modified Duration (MD) = $\frac{D}{1+Y} = \frac{2.81}{1+0.07} = 2.63$ Years

Problem 9.22

Soln

Given:

Face value (M) = Rs. 1000

Annual coupon rate (c) = 12%

Term (n) = 5 years

Yield to Maturity (Y) = 15%

$$\text{Bond Duration } (D) = \frac{1+Y}{Y} - \frac{(1+Y) + n(c-Y)}{c[(1+Y)^n - 1] + Y}$$

$$= \frac{1+0.15}{0.15} - \frac{(1+0.15) + 5(0.12-0.15)}{0.12[(1+0.15)^5 - 1] + 0.15}$$

$$= 7.67 - 3.68$$

$$= 3.99 \text{ years}$$

Problem 9.25

Soln

Given:

Yield To Maturity (Y) = 10%

Face value (M) = Rs. 1000

Coupon rate (c) = 8%. (semi-annually)

Maturity (n) = 3 Yrs

$$D = \frac{1+Y/2}{Y/2} - \frac{(1+Y/2) + 2n(c/2 - Y/2)}{c/2[(1+Y/2)^{2n} - 1] + Y/2}$$

$$= \frac{1+0.10/2}{0.10/2} - \frac{(1+0.10/2) + 2\times3(0.08/2 - 0.10/2)}{0.08/2[(1+0.10/2)^{2\times3} - 1] + 0.10/2}$$

$$= \frac{1+0.05}{0.05} - \frac{(1+0.05) + 2\times3\,(0.04-0.05)}{0.04\left[(1+0.05)^6 - 1\right] + 0.05}$$

$$= 21 - 15.56$$

$$= 5.434 \text{ periods}$$

$$\text{Annual Duration} = \frac{D}{2} = \frac{5.434}{2} = 2.72 \text{ Years}$$

Problem 9.23

Soln

Given: 100 basis points = 1%

Change in interest rate (Δi) = 50 basis point = −0.5%

a. Duration (D) = 8.46 Years

Yield (Y) = 7.5%

$$\text{Change in price } (\Delta p) = -1 \times MD \times \Delta i$$

$$= -1 \times 7.87 \times (-0.5\%)$$

$$= 3.935\%$$

where,

$$MD = \frac{D}{1+Y} = \frac{8.46}{1+0.075} = 7.87 \text{ Yrs}$$

b. Duration (D) = 9.30 Yrs

Yield (Y) = 10%

Change in price (Δp) $= -1 \times MD \times \Delta i$

$= -1 \times 8.45 \times (-0.5\%)$

$= 4.225\ \%$

where,

$$MD = \frac{D}{1+Y} = \frac{9.30}{1+0.10} = 8.45 \text{ Yrs.}$$

c. Duration (D) = 8.75 Yrs

Yield (Y) = 5.75 %

Change in price (Δp) $= -1 \times MD \times \Delta i$

$= -1 \times 8.27 \times (-0.5\%)$

$= 4.135\%$

where,

$$MD = \frac{D}{1+Y} = \frac{8.75 \text{ Yrs}}{1+0.0575} = 8.27 \text{ Yrs}$$

A bond with a Macaulay duration of 9.30 years that is priced to yield 10% should be selected because this bond have higher percentage change in price.

Problem 9.26

Soln

Portfolio Duration (D_p) $= W_A \times D_A + W_B \times D_B + W_I \times D_I + W_D \times D_D$

$= 0.20 \times 4.5 + 0.25 \times 3.0 + 0.25 \times 3.5 + 0.30 \times 2.8$

$= 3.365$ Years.