

**Unit 6: Quantitative Techniques for Economists:  $3 \times 1 + 5 = 8$  Marks**

**6.1 Basic Statistics**

- 6.1.1 Review of Basic Statistics: Definition, Scope, Importance and Limitation of Statistics
- 6.1.2 Collection of Data: Method of Collecting Primary Data and Sources of Secondary Data
- 6.1.3 Standard Deviation, Coefficient of Variation (CV) and Characteristics of Variance
- 6.1.4 Index Number: Price Index Number - Laspeyre's and Paasche's Price Index Number

## MEASURES OF DISPERSION:

### 6.3 Deviations, coefficient of variation and characteristics of variance

Course contents: range, mean deviation, quartile deviation, standard deviation and variance

#### Some Useful Formulae

1. Range (Note: Avoid frequency in case of discrete and continuous series)

$$\text{Range} = L - S$$

$$\text{Coefficient of Range} = \frac{L - S}{L + S}$$

Where, L = largest value of series, S = Smallest value of series

2. Quartile Deviation

$$\text{Quartile Deviation (QD)} = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

3. Mean Deviation (MD) from mean, median and mode

| Series         | Individual                     | Discrete                         | Continuous                       | Coefficient of MD                             |
|----------------|--------------------------------|----------------------------------|----------------------------------|-----------------------------------------------|
| MD from mean   | $\frac{\sum  X - \bar{X} }{n}$ | $\frac{\sum f  X - \bar{X} }{N}$ | $\frac{\sum f  m - \bar{X} }{N}$ | $\frac{\text{MD from mean}}{\bar{X}}$         |
| MD from Median | $\frac{\sum  X - Md }{n}$      | $\frac{\sum f  X - Md }{N}$      | $\frac{\sum f  m - Md }{N}$      | $\frac{\text{MD from Median}}{\text{Median}}$ |
| MD from Mode   | $\frac{\sum  X - Mo }{n}$      | $\frac{\sum f  X - Mo }{N}$      | $\frac{\sum f  m - Mo }{N}$      | $\frac{\text{MD from mode}}{\text{Mode}}$     |

Where mean, median, mode and lower/upper quartile can be calculated by using following formulae:

| Series                                | Individual                                    | Discrete                                                                                                             | Continuous                                                                                                                                               |
|---------------------------------------|-----------------------------------------------|----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Measures                              |                                               |                                                                                                                      |                                                                                                                                                          |
| Arithmetic Mean (AM) or ( $\bar{X}$ ) | $= \frac{\sum X}{N}$                          | $= \frac{\sum fX}{N}$                                                                                                | $= \frac{\sum fm}{N}$                                                                                                                                    |
| Median ( $M_d$ )                      | $= \frac{n+1}{2}^{\text{th}} \text{ item}$    | $= \frac{n+1}{2}^{\text{th}} \text{ item lies on corresponding value of equal or just greater than item in c.f.}$    | $= L + \frac{\frac{N}{2} - cf}{f} \times h$<br>$n/2^{\text{th}} \text{ item lies on corresponding class of Equal or just greater than item in cf}$       |
| Lower Quartile $Q_1$                  | $= \frac{n+1}{4}^{\text{th}} \text{ item}$    | $= \frac{n+1}{4}^{\text{th}} \text{ item lies on corresponding value either equal or greater than the item in c.f.}$ | $= L + \frac{\frac{N}{4} - cf}{f} \times h$<br>$\frac{n}{4}^{\text{th}} \text{ item} = \text{either equal or greater than item lies on class for } Q_1.$ |
| Upper Quartile ( $Q_3$ )              | $= \frac{3(n+1)}{4}^{\text{th}} \text{ item}$ | $\frac{3(n+1)}{4}^{\text{th}} \text{ item lies on corresponding value either equal or greater than the item in CF}$  | $= L + \frac{\frac{3N}{4} - cf}{f} \times h$                                                                                                             |

|                |                                                                                                  |                                                                                                              |
|----------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------|
|                |                                                                                                  | $\frac{3n}{4}$ th item = either equal or greater than item lies on class for $Q_3$ ,                         |
| Mode ( $M_o$ ) | = Repeated value<br>Or, $3M_d - 2\bar{X}$ (from median and mean)<br>= Value of maximum frequency | $= L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times h$<br>( $\Delta_1 = f_1 - f_0$ , $\Delta_2 = f_1 - f_2$ ) |

#### 4. Standard Deviation (SD), Variance and Coefficient

| Series/ Class                               | SD from Actual Mean-                                                           | SD from direct method                                                        |
|---------------------------------------------|--------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| Individual Series                           | $\sigma = \sqrt{\frac{\sum (x - \bar{X})^2}{n}} = \sqrt{\frac{\sum x^2}{n}}$   | $= \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$              |
| Discrete Series                             | $\sigma = \sqrt{\frac{\sum f(x - \bar{X})^2}{N}} = \sqrt{\frac{\sum fx^2}{N}}$ | $= \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$            |
| Continuous Series                           | $\sigma = \sqrt{\frac{\sum f(m - \bar{X})^2}{N}} = \sqrt{\frac{\sum fd^2}{N}}$ | $= \sqrt{\frac{\sum fm^2}{N} - \left(\frac{\sum fm}{N}\right)^2}$            |
| Coefficient and Coefficient Variation of SD | Coefficient of (SD) = $\frac{\sigma}{\bar{X}}$                                 | Coefficient of Variation of SD<br>$CV = \frac{\sigma}{\bar{X}} \times 100\%$ |

Where,

$$x = X - \bar{X}$$

$N = \sum f$  = number of frequency in series

$f$  = frequency of series

$$d = m - \bar{X}$$

$m$  = Mid-value of class

$\bar{X}$  = mean

#### 6.4. Index Number

Course contents: meaning, price index number (simple and weighted), Laspeyres's and Paache's price index numbers

##### Some Useful Formulae

| Calculation of Index Number                                        | Formulae                                                                                                                                                  | Definition of term used in formula                                                                                                                                                                              |
|--------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Simple aggregative method                                          | $P_{01} = \frac{\sum P_1}{\sum P_0} \times 100$                                                                                                           | $P_0$ = price level for base year<br>$P_1$ = price level for current year<br>$\sum P_0$ =<br>Sum of price level for base year<br>$\sum P_1$ =<br>sum of price level for current year                            |
| Average of price relative method<br>(When AM and GM used)          | $P_{01} = \frac{\sum P}{n}$<br>$P_{01} (AM) = \frac{\sum P}{n}$<br>$P_{01} (GM) = \text{Antilog} \left[ \frac{\sum \log P}{N} \right]$                    | $n$ = number of items<br>$P = \frac{P_1}{P_0} \times 100$<br>$\sum P$ = Sum of current year price                                                                                                               |
| Weighted average of price relative method<br>(When AM and GM used) | $P_{01} = \frac{\sum PW}{\sum W}$<br>$P_{01} (AM) = \frac{\sum PW}{\sum W}$<br>$P_{01} (GM) = \text{Antilog} \left[ \frac{\sum W \log P}{\sum W} \right]$ | $P$ = price relative = $\frac{P_1}{P_0} \times 100$<br>$W$<br>= weight of the commodity on base year<br>$\sum W$ =<br>Sum of the weight of commodities<br>$\sum PW$ = Sum of the weight $\times$ price relative |
| Laspeyre's price index method                                      | $P_{01} L = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$                                                                                                 | $P_0$ = price of base year<br>$P_1$ = price level for current year                                                                                                                                              |
| Paasche's price index method                                       | $P_{01} (P) = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$                                                                                               | $q_0$ = quantity of base year<br>$q_1$ = quantity of current year                                                                                                                                               |

#### Numerical Questions: (1 mark)

20. Find the value of K if the mean of given data is 9.

Income (in Rs.): 8, 4, 12, K, 10, 13

Solution: Given

$$\sum X = 8 + 4 + 12 + K + 10 + 13 = 47 + K$$

$$N = 6,$$

$$\text{Mean } (\bar{X}) = ?$$

We have,

$$\bar{X} = \frac{\sum X}{N}$$

$$\text{Or, } 9 = \frac{47+K}{6}$$

$$\text{Or, } 47 + K = 54$$

$$K = 54 - 47$$

$$K = 7$$

$\therefore$  The value of K is 7.

21. Find the median from the following data.

Length (in cm): 8, 4, 12, 7, 10, 13, 9, 6

Solution,

Arranging the given data in the ascending order: 4, 6, 7, 8, 9, 10, 12, 13

Here,

$$N = 7$$

$$\text{Median (Md)} = ?$$

We have,

$$Md = \frac{n+1}{2} \text{th item} = \frac{7+1}{2} = 4$$

$$4^{\text{th}} \text{ item in data series} = 8$$

$$\therefore Md = 8$$

22. Find the mode from the following data.

Marks (X): 4, 5, 6, 3, 5, 7, 9, 5,

Soln:

We have,

Mode is the maximum repeated number in series.

$$\therefore \text{Mode} = 5$$

23. If the difference and sum of first (lower) quartile and third (upper) quartile are 20 and 48 respectively. Find the coefficient of quartile deviation.

Soln: Given

Difference of upper and lower quartile ( $Q_3 - Q_1$ ) = 20

Sum of upper and lower quartile ( $Q_3 + Q_1$ ) = 48

Coefficient of Quartile Deviation (C.QD) = ?

We have

$$\text{C.QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{20}{48} = 0.42$$

$$\therefore \text{Coefficient of quartile deviation} = 0.42$$

24. If coefficient of variance (CV) is 0.39 and standard deviation is 7.48, find the mean.

Soln: Given

$$CV = 0.39$$

$$SD(\sigma) = 7.48$$

$$\bar{X} = ?$$

We have

$$CV = \frac{\sigma}{\bar{X}}$$

$$\text{Or, } 0.39 = \frac{7.48}{\bar{X}}$$

$$\text{Or, } \bar{X} = \frac{7.48}{0.39} = 19.18$$

$$\therefore \text{Mean} = 19.18$$

25. If the mode of asymmetrical distribution is 370 and median is 300. Find the mean.

Soln: Given

$$\text{Mode (Mo)} = 370$$

$$\text{Median (Md)} = 300$$

$$\text{Mean } (\bar{X}) = ?$$

In case of asymmetrical distribution of data series, we have

$$Mo = 3 Md - 2 \bar{X}$$

$$\text{Or } 370 = 3 \times 300 - 2 \bar{X}$$

Or  $2\bar{X} = 900 - 370$

$\bar{X} = \frac{530}{2} = 265$

$\therefore \text{Mean} = 265$

26. Find the range and coefficient of range from the following data.

45, 20, 25, 15, 40, 50, 35, 30

Solution,

Data in ascending order: 15, 20, 25, 30, 35, 40, 45, 50

Here,

Largest item (L) = 50

Smallest item (S) = 15

Range = ?

Coefficient of Range = ?

We have,

Range = L - S = 50 - 15 = 35

$\therefore \text{Range} = 35$

Similarly,

Coefficient of Range =  $\frac{L-S}{L+S} = \frac{50-15}{50+15} = \frac{35}{65} = 0.54$

$\therefore \text{Coefficient of Range} = 0.54,$

### Short answers (5 marks)

27. If the coefficient of variance of factory A and B are 0.78 and 0.96 respectively, then find out:

- In which factory is there greater variation in the distribution of wage per employee?
- Which factory shows more consistency?

Solution: a.

$CV_A = 0.78$

$CV_B = 0.96$

Coefficient of variance of factory A =  $0.78 \times 100\% = 78\%$

Coefficient of variance of factory B =  $0.96 \times 100\% = 96\%$

Since, coefficient of variation is more of factory B, there is greater variation in the distribution of wage per employee.

Solution b: Since the coefficient of variation of factory A is less than of coefficient of variation of factory B. So, factory A has more consistency than factory B.

28. Find the quartile deviation and coefficient of quartile deviation from the following data series.

| Wages (Rs.)    | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|----------------|------|-------|-------|-------|-------|-------|-------|
| No. of workers | 5    | 3     | 7     | 5     | 10    | 3     | 2     |

Solution: Calculation of Quartile Deviation (QD) and

Coefficient of Quartile Deviation (CQD)

| Wages (X) | No. of frequency (f) | Cumulative frequency (CF) |
|-----------|----------------------|---------------------------|
| 0-10      | 5                    | 5                         |
| 10-20     | 3                    | 8                         |
| 20-30     | 7                    | 15                        |
| 30-40     | 5                    | 20                        |
| 40-50     | 10                   | 30                        |
| 50-60     | 3                    | 33                        |

### Price Index Number

1. Find the price index number by using simple aggregative method, taking 2020 as the current year and 2019 as base year and interpret the result.

| Commodities:        | Tomato | Potato | Wheat | Paddy |
|---------------------|--------|--------|-------|-------|
| Price in 2019 (Rs.) | 50     | 70     | 100   | 67    |
| Price in 2020 (Rs.) | 100    | 130    | 80    | 150   |

Solution: Calculation of Simple aggregative price index

| Commodities | Price in 2019 (P <sub>0</sub> ) | Price in 2020 (P <sub>1</sub> ) |
|-------------|---------------------------------|---------------------------------|
| Tomato      | 50                              | 100                             |
| Potato      | 70                              | 130                             |
| Wheat       | 100                             | 80                              |
| Rice        | 67                              | 150                             |
|             | $\Sigma P_0 = 287$              | $\Sigma P_1 = 460$              |

$$P_{01} = \frac{\Sigma P_1}{\Sigma P_0} \times 100$$

We know,

$$\text{Here, simple aggregative price index (P}_{01}) = \frac{\Sigma P_1}{\Sigma P_0} \times 100 = \frac{460}{287} \times 100 = 160.28\%$$

**Interpretation:** The general price level has been increased by  $160.28 - 100 = 60.28\%$  in 2020 in comparison to 2019

2. From the given data, find the price index number for the year 2020 taking 2019 as a base year by using average of price relative method.

| Commodities | Price in 2019 (Rs.) | Price in 2020 (Rs.) |
|-------------|---------------------|---------------------|
| A           | 6                   | 10                  |
| B           | 2                   | 2                   |
| C           | 4                   | 6                   |
| D           | 10                  | 12                  |
| E           | 8                   | 12                  |

Solution: Calculation of price index by using average of price relative method

| Commodities | Price in 2019 (P <sub>0</sub> ) | Price in 2020 (P <sub>1</sub> ) | $P = \frac{P_1}{P_0} \times 100$ |
|-------------|---------------------------------|---------------------------------|----------------------------------|
| A           | 6                               | 10                              | 166.67                           |
| B           | 2                               | 2                               | 100                              |
| C           | 4                               | 6                               | 150                              |
| D           | 10                              | 12                              | 120                              |
| E           | 8                               | 12                              | 150                              |
| N = 5       |                                 |                                 | $\Sigma P = 686.67$              |

We know, for simple average of price relative index number

$$P_{01} = \frac{\Sigma P}{n} = \frac{686.67}{5} = 137.33\%$$

**Interpretation:** The price has been increased by  $137.33 - 100 = 37.33\%$  in 2020 in comparison to 2019.

3. Calculate the price index from the following data by using weighted average of price relative for the year 2020.

| Commodities | 2019  |          | 2020  |          |
|-------------|-------|----------|-------|----------|
|             | Price | Quantity | Price | Quantity |
| A           | 10    | 100      | 20    | 120      |
| B           | 11    | 150      | 25    | 180      |
| C           | 12    | 180      | 30    | 210      |
| D           | 14    | 200      | 40    | 250      |

**Solution:** Computation for Price Index using weighted average of price relative

| Commodities | $p_0$ | $p_1$ | $q_0$ | $q_1$ | $P = \frac{p_1}{p_0} \times 100$ | $W = p_0 q_0$     | PW                    |
|-------------|-------|-------|-------|-------|----------------------------------|-------------------|-----------------------|
| A           | 10    | 20    | 100   | 120   | 200                              | 1000              | 200000                |
| B           | 11    | 25    | 150   | 180   | 227.2                            | 1650              | 374880                |
| C           | 12    | 30    | 180   | 210   | 250                              | 2160              | 540000                |
| D           | 14    | 40    | 200   | 250   | 285.7                            | 2800              | 799960                |
| Total       |       |       |       |       |                                  | $\Sigma W = 7610$ | $\Sigma PW = 1914840$ |

$$P_{01} = \frac{\Sigma PW}{\Sigma W} = \frac{1914840}{7610} = 251.6\% \quad (\text{If weight is not given, } W = P_0 Q_0)$$

**Interpretation:** The price has been increased by  $251.6 - 100 = 151.6\%$  in 2020 in comparison to 2019.

4. Calculate the price index number by using Laspeyre's method and Paasche's method from the following data and interpret the result.

| Commodities | Base year 2021 |          | 2023  |          |
|-------------|----------------|----------|-------|----------|
|             | Price          | Quantity | Price | Quantity |
| A           | 2              | 20       | 5     | 15       |
| B           | 4              | 4        | 8     | 5        |
| C           | 1              | 10       | 2     | 12       |
| D           | 5              | 5        | 10    | 6        |

**Solution:** Computation of price index numbers by using Laspeyre's and Paasche's method

| Commodities | $p_0$ | $q_0$ | $p_1$ | $q_1$ | $p_0 q_0$             | $p_1 q_0$              | $p_0 q_1$             | $p_1 q_1$              |
|-------------|-------|-------|-------|-------|-----------------------|------------------------|-----------------------|------------------------|
| A           | 2     | 20    | 5     | 15    | 40                    | 100                    | 30                    | 75                     |
| B           | 4     | 4     | 8     | 5     | 16                    | 32                     | 20                    | 40                     |
| C           | 1     | 10    | 2     | 12    | 10                    | 20                     | 12                    | 24                     |
| D           | 5     | 5     | 10    | 6     | 25                    | 50                     | 30                    | 60                     |
| Total       |       |       |       |       | $\Sigma p_0 q_0 = 91$ | $\Sigma p_1 q_0 = 202$ | $\Sigma p_0 q_1 = 92$ | $\Sigma p_1 q_1 = 199$ |

For Laspeyre's price index number, we have

$$P_{01}(L) = \frac{\Sigma p_1 q_0}{\Sigma p_0 q_0} \times 100 = \frac{202}{91} \times 100 = 221.29\%$$

**Interpretation:** The price has been increased by  $221.29 - 100 = 121.29\%$  in 2023 in comparison to 2021

For Paasche's price index number, we have

$$P_{01}(P) = \frac{\Sigma p_1 q_1}{\Sigma p_0 q_1} \times 100 = \frac{199}{92} \times 100 = 216.36\%$$

**Interpretation:** The price has been increased by  $216.36 - 100 = 116.36\%$  in 2023 in comparison to 2021

Best of Luck!!!!